Name: ____________________

Drawing Equipotential Surfaces

Working in small groups (2 or 3 people), solve as many of the problems below as possible. Try to resolve questions within the group before asking for help. Each group member should then write up solutions in their own words.

Start with a Simple(r) Case: The electrostatic potential due to a particle with charge q is:

$$V(r) = \frac{kq}{r}$$

where k is the electrostatic constant and r is the distance from the particle.

On your whiteboard, identify all the points with the same value of potential around a single point charge. Repeat for several different values of potential.

- What shapes have you drawn?
- If you wanted the difference in potential represented by the shapes to be equal, how are they spaced?

(1) **Alternative:** I like to do this question as a small whiteboard question, where every student draws their answer on an individual whiteboard.

(2) **Whole Class Discussion:** This is a great place for a whole class discussion in the middle of an activity. It is important that students understand that the distance between surfaces increases as you move away from the particle, that symmetry is a very useful thing to notice, and that the surfaces are in fact nested spheres in space and not just the circles on the board.

(3) **Discussion Item - Inverse Function:** It is helpful to plot an inverse function and to show that equally spaced intervals for potential correspond to increasing separation.

Add Complexity: Draw equipotential surfaces for the potential due to 4 particles with equal, positive charge arranged in a square.

(1) **Prep:** For the next part ("Go"), I like to use a template to draw the position of the particles so that the scale matches the dry-erasable surface. Therefore, I also do the same here (even though there is no surface).

(2) **Discussion Item - Forces vs. Potentials:** Students tend to want to make arguments about forces rather than potentials. They will therefore tend to conclude incorrectly that the potential at the center of the square is zero. Remind students that force is related how the potential changes, and that the derivative of a function can be zero at a location where the value of the function is not zero.

(3) **Discussion Item - Far away from the source** Students usually quickly realize that, far away, the square looks like a point charge and the surfaces are spheres.

(4) **Discussion Item - Distance and fall off** Students will need to think about how the potential falls off faster nearer the charge and is essentially constant far away from the charge.

(5) **Thing to Notice:** We have seen students try to draw 3D perspective drawings of a graph of V. Everyone has trouble interpreting these drawings, including the drawer. It is worth talking to students about what they are trying to represent and then show them the Mathematica notebook.

(6) **Whole Class Discussion:** This is also a good place to interrupt the activity and have students discuss how they chose the shape and separation of the surfaces before working on "Go".

(7) **Mathematica Notebook:** After this section, I find it useful to demonstrate the Mathematica notebook for this activity and discuss the many different ways to represent a 3D scalar field.

Examine a New Case: Repeat for a quadrupole - 2 positively charged particles and 2 negatively charged particles arranged in a square, with "like" charged particles on opposite corners.

(1) **Prep:** I like to use a template to draw the position of the particles so that the scale matches the dry-erasable surface. I recommend giving the students the surface after the groups have been working for a few minutes (but not too long).

(2) **Discussion Item - Surfaces for V=0:** If students are floundering, it is helpful to direct their attention to places where the potential is zero. It bothers some students that the equipotential sphere at infinity intersects the V=0 symmetry planes. Some students remember from their intro course or from a math course that equipotential surfaces/level curves should not intersect.

(3) **Discussion Item - Sign of the charge** This example is more difficult than the previous one because now students have to take into consideration the sign of the potential when adding them together.

(4) **Discussion Item - using multiple representations** We have observed students using the Mathematica notebook and the surface to "look up the answer" and then try to explain it. We encourage this kind of behavior as a way for the students to get themselves unstuck.

Activity Evaluation

What was the main point of this activity?

Describe one thing you understand as a result of this activity.

Describe one thing that is confusing after completing this activity.