AMPERE'S LAW

- A steady current is flowing parallel to the axis through an infinitely long cylindrical shell of inner radius a and outer radius b. Each group is assigned one of the current densities given below: (In each case, α and k are constants with appropriate units.)

  $|\vec{J}| = \alpha\, r^3$.

  $|\vec{J}| = \alpha\, \frac{\sin kr}{r}$.

  $|\vec{J}| = \alpha\, e^{kr^2}$.

  $|\vec{J}| = \alpha\, \frac{e^{kr}}{r}$.

  For your group's case, answer each of the following questions:

1. What units do α and k have?

   This is a good opportunity to point out that the arguments of special functions must be dimensionless (link to further explanation) which suffices to determine the dimensions of k. The dimensions of k are different in each case. On the other hand, since the functions in the last three cases are inherently dimensionless, the dimensions of α are the same for these three cases. The function in the first case are (length)^3, so the dimensions of α are different for this case.

2. Find the total current flowing through the wire.

   The total current flowing through the wire is different in each case, thus making it slightly more difficult to compare the graphs in question 4 across different cases. Of course, it would be possible to put a multiplicative constant out in front of each case, that would adjust the currents to agree. But there are many reasons not to do this: students are confused generally by parameters (link to further explanation); the real world never comes with problems adjusted to be nice like this; this is one of those places where the discussion that arises from a problem that is not stated perfectly smoothly actually aids the students' comprehension–where on the graph does the total current appear?

   The original wording for this problem was: "find the total current through a cross section of the wire perpendicular to the axis." My, I used to give a lot away! Activities earlier in the course (link to a discussion of current or to the AJP Ampere's law paper) will have helped the students understand that total current is a flux. This is a great chance to get them to try to connect that knowledge. If they are not sure of the surface to use, then, that makes a perfect class discussion!

3. Use Ampere's Law and symmetry arguments to find the magnetic field at each of the three radii given below:

   $r_1 < a$

   $a < r_2 < b$

   $r_3 > b$

   This is a straighforward Ampere's Law calculation. But, watch carefully for the following two behaviors:

   1) In part a), the students often assume that thing that they are trying to prove, namely that the magnetic field is zero inside the cylinder. Typically, they are using memory from an earlier class. Acknowledge that their memory is correct, but point out to them that they are nevertheless expected *to prove the result here. Sometimes, however, they get this result as a misapplication of a symmetry argument. Listen closely.*

2) Students tend to mimic the words "from symmetry" without really understanding what that means. They will draw the contribution to the magnetic field from two symmetrically placed current bits. Check, first, that they are getting their right hand rule right and that they really get those two bits canceling (or not canceling) correctly. Then ask them to think about contributions from other points and make sure they can get those correct. A few students can make this argument really work, but then make them right it down. Most students see that they can't really manage it and begin to appreciate the argument given in the AJP Ampere paper (link).

4. For $\alpha = 1$, $k = 1$, sketch the magnitude of the magnetic field as a function of r.

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