Recorder:

Task Master: \_\_\_\_\_

Cynic: \_

Other:

## ACCELERATION

Working in small groups (3 or 4 people), solve as many of the problems below as possible. Try to resolve questions within the group before asking for help. The Recorder is responsible for writing up the group's results and turning it in. Show your work! Full credit will only be given if your answer is supported by calculations and/or explanations as appropriate.

## 0. WARMUP

In rectangular coordinates, the natural unit vectors are  $\{\hat{i}, \hat{j}\}$ , which point in the direction of increasing x and y, respectively. Similarly, in polar coordinates the natural unit vectors are  $\hat{r}$ , which points in the direction of increasing r, and  $\hat{\phi}$ , which points in the direction of increasing  $\phi$ .



Sketch  $\hat{r}$  and  $\hat{\phi}$  at several points on the unit circle.

## 1. CIRCLES

The *unit tangent vector* to a parametric curve is the unit vector tangent to the curve which points in the direction of increasing parameter. The *principal unit normal vector* to a parametric curve is the unit vector perpendicular to the curve "in the direction of bending", which is the direction of the *derivative* of the unit tangent vector.

- (a) Consider the parametric curve  $\vec{r} = 3\cos\phi\,\hat{i} + 3\sin\phi\,\hat{j}$  with  $\phi \in [0, 2\pi]$ . Calculate the unit tangent vector  $\hat{T}$  and the principal unit normal vector  $\hat{N}$  for this curve in terms of  $\hat{i}$  and  $\hat{j}$ .
- (b) Consider a circle of radius 3 centered at the origin. Determine the unit tangent vector  $\hat{T}$  and the principal unit normal vector  $\hat{N}$  for this curve in terms of  $\hat{r}$  and  $\hat{\phi}$ .
- (c) Compare your answers from parts (a) and (b).