

## Group Activity 2: Acceleration

### I Essentials

#### (a) Main ideas

- Geometric introduction of  $\hat{\mathbf{r}}$  and  $\hat{\boldsymbol{\phi}}$ .
- Geometric introduction of unit tangent and normal vectors.

#### (b) Prerequisites

- The position vector  $\vec{\mathbf{r}}$ .
- The derivative of the position vector is tangent to the curve.

#### (c) Warmup

See the prerequisites. It is possible to briefly introduce these ideas immediately preceding this activity.

#### (d) Props

- whiteboards and pens

#### (e) Wrapup

- Emphasize that  $\hat{\mathbf{r}}$  and  $\hat{\boldsymbol{\phi}}$  do not live at the origin! Encourage students to use the figure provided, which may help alleviate this confusion.
- Point out to the students that  $\hat{\mathbf{r}}$  and  $\hat{\boldsymbol{\phi}}$  are defined everywhere (except at the origin), whereas  $\hat{\mathbf{T}}$  and  $\hat{\mathbf{N}}$  are properties of the curve. It is only on circles that these two notions coincide;  $\hat{\mathbf{r}}$  and  $\hat{\boldsymbol{\phi}}$  are adapted to round problems, and circles are round! Symmetry is important.
- Emphasize that  $\{\hat{\mathbf{r}}, \hat{\boldsymbol{\phi}}\}$  can be used as a basis (except at the origin). Point out to the students that their answer to the last problem gives them a formula expressing  $\hat{\mathbf{r}}$  and  $\hat{\boldsymbol{\phi}}$  in terms of  $\hat{\mathbf{i}}$  and  $\hat{\mathbf{j}}$ . When comparing these basis vectors, they should all be drawn with their tails at the same point.

## II Details

We have had success helping students master the idea of “direction of bending” by describing the curve as part of a pickle jar; the principal unit normal vector points at the pickles!

### (a) In the Classroom

The easiest way to find  $\hat{N}$  is to use the dot product to find vectors orthogonal to  $\hat{T}$ , then normalize. Students must then use the “direction of bending” criterion to choose between the two possible orientations.

Finding  $\hat{N}$  in this way requires the student to give names to the its unknown components. This is a nontrivial skill; many students will have trouble with this. This is a good example of the general skill discussed in Section 11.1.

### (b) Subsidiary ideas

- Dividing any vector by its length yields a unit vector.
- Using the dot product to find vectors perpendicular to a given vector.

### (c) Homework

- Some students will not be comfortable unless they work out the components of  $\hat{r}$  and  $\hat{\phi}$  with respect to  $\hat{i}$  and  $\hat{j}$ . Let them.

### (d) Essay questions (none yet)

### (e) Enrichment

- What units does a unit vector have? Do  $\hat{r}$  and  $\hat{\phi}$  have the same units?