White board activity

- Find <Sx>
- <Sy>
- And <Sz>
- For the general state $|\psi(t)\rangle \doteq e^{-i\frac{\omega_0 t}{2}} \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i(\omega_0 t+\phi)} \sin\frac{\theta}{2} \end{pmatrix}$

Expectation Value of Spin Angular Momentum:

$$\langle S_z \rangle = \left(+\frac{\hbar}{2} \right) P(+) + \left(-\frac{\hbar}{2} \right) P(-) = \langle \psi(t) | S_z | \psi(t) \rangle$$

$$= e^{i\frac{\omega_0 t}{2}} \begin{pmatrix} \cos\frac{\theta}{2} & e^{-i(\phi+\omega_0 t)} \sin\frac{\theta}{2} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} e^{-i\frac{\omega_0 t}{2}} \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i(\phi+\omega_0 t)} \sin\frac{\theta}{2} \end{pmatrix}$$

$$= \frac{\hbar}{2} \begin{pmatrix} \cos\frac{\theta}{2} & e^{-i(\phi+\omega_0 t)} \sin\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} \\ -e^{i(\phi+\omega_0 t)} \sin\frac{\theta}{2} \end{pmatrix}$$

$$= \frac{\hbar}{2} \left[\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} \right] = \frac{\hbar}{2} \cos\theta$$

$$\left\langle S_x \right\rangle = \left\langle \psi(t) \middle| S_x \middle| \psi(t) \right\rangle$$

$$= e^{i\frac{\omega_0 t}{2}} \begin{pmatrix} \cos\frac{\theta}{2} & e^{-i(\phi+\omega_0 t)} \sin\frac{\theta}{2} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} e^{-i\frac{\omega_0 t}{2}} \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i(\phi+\omega_0 t)} \sin\frac{\theta}{2} \end{pmatrix}$$

$$= \frac{\hbar}{2} \begin{pmatrix} \cos\frac{\theta}{2} & e^{-i(\phi+\omega_0 t)} \sin\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} e^{i(\phi+\omega_0 t)} \sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}$$

$$= \frac{\hbar}{2} \cos\frac{\theta}{2} \sin\frac{\theta}{2} \left[e^{i(\phi+\omega_0 t)} + e^{-i(\phi+\omega_0 t)} \right] = \frac{\hbar}{2} \sin\theta \cos(\phi + \omega_0 t)$$

$$\left\langle S_y \right\rangle = \left\langle \psi(t) \middle| S_y \middle| \psi(t) \right\rangle$$

$$= e^{i\frac{\omega_0 t}{2}} \begin{pmatrix} \cos\frac{\theta}{2} & e^{-i(\phi+\omega_0 t)} \sin\frac{\theta}{2} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} e^{-i\frac{\omega_0 t}{2}} \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i(\phi+\omega_0 t)} \sin\frac{\theta}{2} \end{pmatrix}$$

$$= \frac{\hbar}{2} \begin{pmatrix} \cos\frac{\theta}{2} & e^{-i(\phi+\omega_0 t)} \sin\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} -ie^{i(\phi+\omega_0 t)} \sin\frac{\theta}{2} \\ i\cos\frac{\theta}{2} \end{pmatrix}$$

$$= \frac{\hbar}{2} \cos\frac{\theta}{2} \sin\frac{\theta}{2} \left[-ie^{i(\phi+\omega_0 t)} + ie^{-i(\phi+\omega_0 t)} \right] = \frac{\hbar}{2} \sin\theta \sin(\phi + \omega_0 t)$$

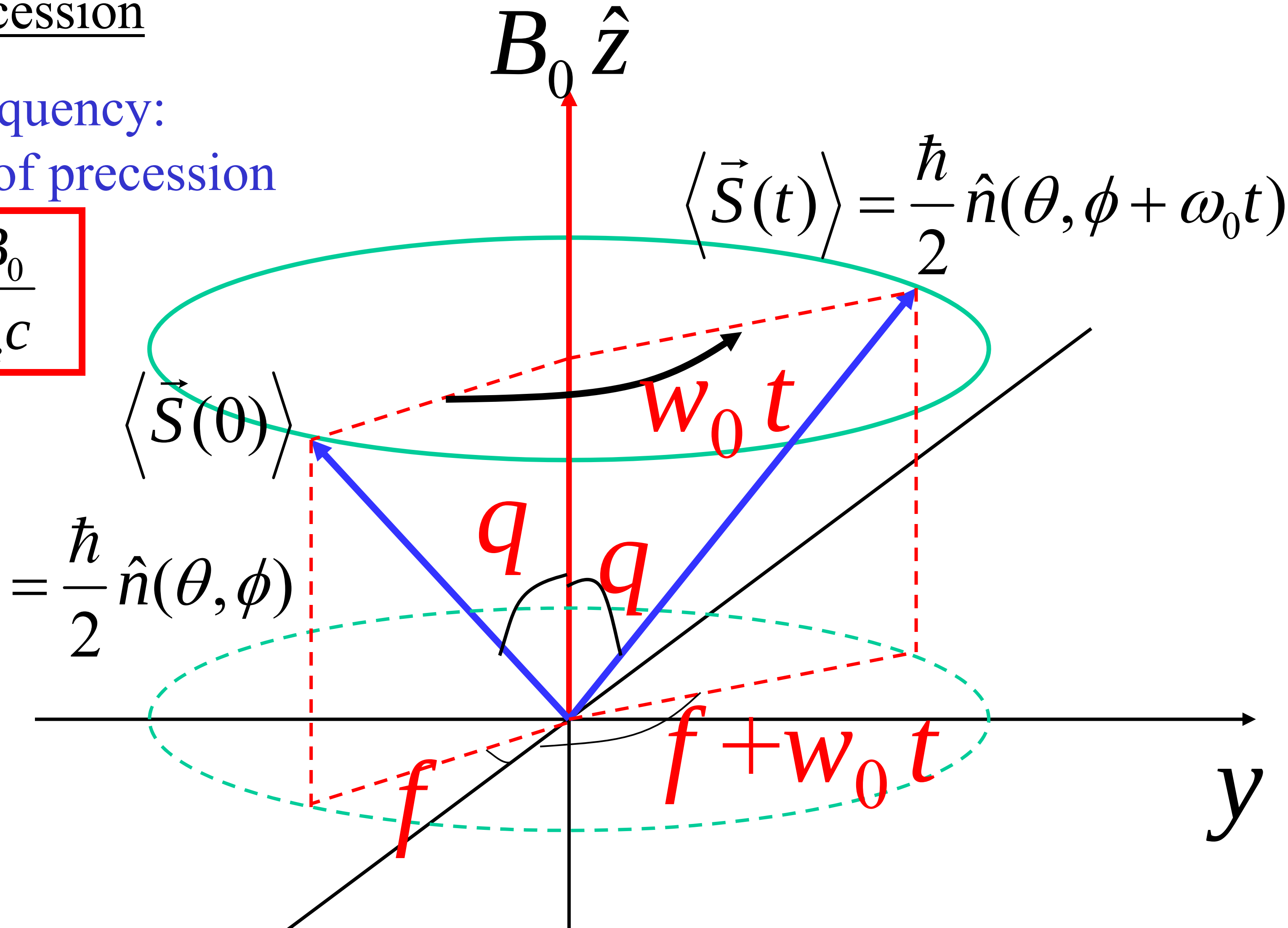
$$\left\langle \vec{S}(t) \right\rangle = \left\langle S_x \right\rangle \hat{x} + \left\langle S_y \right\rangle \hat{y} + \left\langle S_z \right\rangle \hat{z}$$

$$= \frac{\hbar}{2} \left[\hat{x} \sin\theta \cos(\phi + \omega_0 t) + \hat{y} \sin\theta \sin(\phi + \omega_0 t) + \hat{z} \cos\theta \right] = \frac{\hbar}{2} \hat{n}(\theta, \phi + \omega_0 t)$$

Spin Precession

Larmor frequency:
frequency of precession

$$\omega_0 = \frac{eB_0}{m_e c}$$



$$\langle \vec{S}(t) \rangle = \langle S_x \rangle \hat{x} + \langle S_y \rangle \hat{y} + \langle S_z \rangle \hat{z}$$

$$= \frac{\hbar}{2}\left[\hat{x}\sin\theta\cos(\phi + \omega_0 t) + \hat{y}\sin\theta\sin(\phi + \omega_0 t) + \hat{z}\cos\theta\right] = \frac{\hbar}{2}\hat{n}(\theta, \phi + \omega_0 t)$$

$$\left\langle \vec{S}(t) \right\rangle = \langle S_x \rangle \hat{x} + \langle S_y \rangle \hat{y} + \langle S_z \rangle \hat{z}$$

$$= \frac{\hbar}{2}\left[\hat{x}\sin\theta\cos(\phi + \omega_0 t) + \hat{y}\sin\theta\sin(\phi + \omega_0 t) + \hat{z}\cos\theta\right] = \frac{\hbar}{2}\hat{n}(\theta, \phi + \omega_0 t)$$

What is <S(t)>:

- if we start in a |+>x state?
- If we start in a |+>y state?
- If we start in a |+> state?

What does this mean?