

SPINS Lab 2*Instructor version*

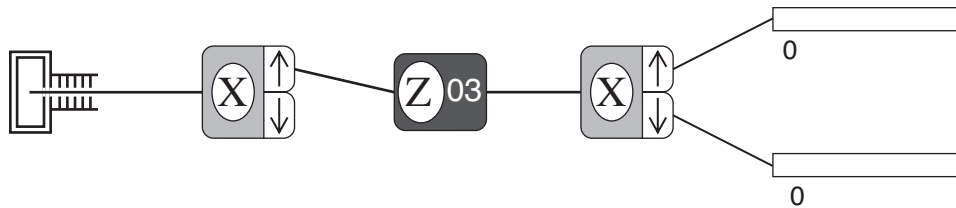
1. Start the SPINS program and choose Unknown #1 under the Initialize menu. This causes the atoms to leave the oven in a definite quantum state, which we call $|\psi_1\rangle$. Now measure the six probabilities $|\langle\phi|\psi_1\rangle|^2$, where $|\phi\rangle$ corresponds to spin up or spin down along the three axes. Fill in the table for $|\psi_1\rangle$ on the worksheet. Use your results to figure out what $|\psi_1\rangle$ is, using the following procedure (even though it may be obvious what the unknown state is for the first few cases, follow the procedure as practice for the harder cases to follow):
- i) Assume that we want to write the unknown state vector in terms of the $|\pm\rangle$ basis, i.e. $|\psi_1\rangle = a|+\rangle + b|-\rangle$, where a and b are complex coefficients. We thus must use the data to find the values of a and b .
 - ii) Use the measured probabilities of spin up and spin down along the z -axis first. This will allow you to determine the magnitudes of a and b . Since an overall phase of the state vector has no physical meaning, we follow the convention that the coefficient of $|+\rangle$ (i.e. a) is chosen to be real and positive. If we write $b = |b|e^{i\phi}$, then you have determined everything except the phase ϕ .
 - iii) Use the measured probabilities of spin up and spin down along the x -axis to provide information about the phase of b . In some cases, this will be unambiguous, in other cases not.
 - iv) If needed, use the measured probabilities of spin up and spin down along the y -axis.
 - v) Confirm your results by entering your calculated coefficients into the table for User State and running the experiment.

Repeat this exercise for Unknown #2 ($|\psi_2\rangle$), Unknown #3 ($|\psi_3\rangle$), and Unknown #4 ($|\psi_4\rangle$). Design an experiment to verify your results (Hint: recall the general spin-1/2 state vector can be written as $|+\rangle_n = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}e^{i\phi}|-\rangle$). (Note that this is Problem 1.17.)

1. This experiment and the homework problem it leads to are not commonly encountered in Quantum Mechanics courses. The students must take data and then use the results to deduce the quantum state vectors that led to those results. The unknown state vectors are preprogrammed into the SPINS program, but can be changed if you desire. It is important that the student follow the prescribed method for finding the state vectors. The first two unknowns ($|\psi_1\rangle = |+\rangle$ and $|\psi_2\rangle = |-\rangle_y$) are so simple that the students can write down the solutions by inspection, but they will then typically have extreme difficulty with the third unknown since they have not gained experience in how to solve for the

unknown state. The order of solution (use Z, then X, then Y data) is chosen to coordinate with the unknowns and give the students practice in the early, easy cases. Students are guided to use the amplitude and phase notation for complex numbers since this makes it easy to associate states with the general state $|+\rangle_n = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}e^{i\phi}|-\rangle$ and then to use the aligned detector to verify their results. The angle ϕ in the unknown states 3&4 is chosen to be greater than π so the students must wrestle with the fact that the inverse trig functions on their calculators do not tell the whole story.

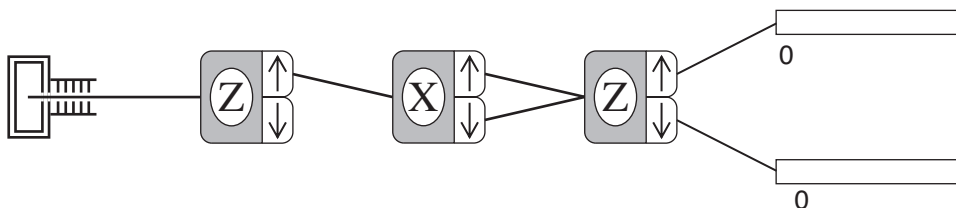
2. Make another unknown state using the following setup with a magnet in between two Stern-Gerlach analyzers:



Consider the magnet for now as a black box that transforms the input $|+\rangle_x$ state into a new state $|\psi\rangle$. Use Random as the initial state and set the strength of the magnet to a number from 1-20 corresponding to the position of your computer in the lab. Use the last analyzer to measure the probabilities for the state $|\psi\rangle$ to have the six possible spin projections along the three axes. Keep the first Stern-Gerlach analyzer and the middle magnet oriented as shown in the figure. Fill in the table on the worksheet and deduce the state $|\psi\rangle$, in terms of the $|\pm\rangle$ basis. Design an experiment to verify your results. From the results of the whole class, can you figure out what the magnet does?

2. This exercise is meant to give more practice in finding the unknown states and to provide a lead-in to Lab 4 later when they will determine what the magnetic field does. For now it is simply a black box and they have no inkling what it might do. Typically we ask the students to pool their results (usually later in the course) and use the information to deduce what the field does. Since the field rotates the state by 10 degrees for each digit on the field icon, assigning numbers up to 18 to a group of students provides enough data to give a complete picture of the effect of the field.

3. Make an interferometer as shown:



Use Random as the initial state. Measure the relative probabilities of spin up and spin down after the final SG device. Do this for the three cases where (1) the spin up beam from the middle SG device is used, (2) the spin down beam from the middle SG device is used, and (3) both beams from the middle SG device are used (as the figure shows). Put your experimental results in the table on the worksheet. Repeat for the case where the spin down state is used from the first SG device.

Now calculate the theoretically expected probabilities and fill in the theory part of the worksheet. When both beams from the middle SG device are used, the input state to the last SG device (call it $|\psi\rangle$) is a combination (superposition) of the spin up and spin down states with respect to the axis of the middle SG device. To calculate $|\psi\rangle$ properly, use the projection postulate (5th postulate), which says that the ket after a measurement is the normalized projection onto the subspace corresponding to the measured results. For the case where both spin up and spin down results are obtained (because both beams are sent to the next device), the projection operator is $P = |+\rangle_x \langle +| + |-\rangle_x \langle -|$. For the case where the input to the middle SG device is $|+\rangle$, the projection operation yields

$$P|+\rangle = |+\rangle_x \langle +| + |-\rangle_x \langle -| |+\rangle = \frac{1}{\sqrt{2}} (|+\rangle_x \langle +| + |-\rangle_x \langle -|) |+\rangle_x = \frac{1}{2} (|+\rangle_x \langle +| + |-\rangle_x \langle -|) |+\rangle_x + \frac{1}{2} (|+\rangle_x \langle +| + |-\rangle_x \langle -|) |-\rangle_x.$$

To find $|\psi\rangle$, just normalize this new ket.

3. This interferometer is Experiment 4 from the text (sec. 1.1.4 on p. 8). The three experiments they do here are equivalent to those shown in Fig. 1.6 a-c on p. 9 of the text. To compare their results with theory, the students are asked to use the projection postulate, which allows them to find the state after a measurement. In this problem, the calculations are simple, but it provides good exercise for the more difficult spin-1 interferometer in Lab 3.

4. Using the interferometer shown above, select Watch under the Control menu. This feature causes a light to flash at the port where an atom exits. The program now allows only the Go mode. Run the experiment and notice how your results compare to the results obtained above. Explain the new results.

4. This experiment is akin to a Heisenberg microscope, where information is obtained about which path particles take through a double slit interferometer. In that case, the fringes are destroyed. Here, the coherence is destroyed and the probability of the last measurement is 50/50.

SPINS Lab 2 WorksheetUnknown $|\psi_1\rangle$

Probabilities	Axis		
Result	x	y	z
spin up \uparrow			
spin down \downarrow			

Unknown $|\psi_2\rangle$

Probabilities	Axis		
Result	x	y	z
spin up \uparrow			
spin down \downarrow			

Unknown $|\psi_3\rangle$

Probabilities	Axis		
Result	x	y	z
spin up \uparrow			
spin down \downarrow			

Unknown $|\psi_4\rangle$

Probabilities	Axis		
Result	x	y	z
spin up \uparrow			
spin down \downarrow			

State $|\psi\rangle$ made with magnet. $B = \text{_____}$

Probabilities	Axis		
Result	x	y	z
spin up \uparrow			
spin down \downarrow			

Spin 1/2 Interferometer

Beams used		Experiment		Theory	
SG1	SG2	\mathcal{P}_+	\mathcal{P}_-	\mathcal{P}_+	\mathcal{P}_-
\rightarrow \rangle	\rightarrow \rangle_x				
	\leftarrow \rangle_x				
	\rightarrow $\rangle_x, \leftarrow \rangle_x$				
\leftarrow \rangle	\rightarrow \rangle_x				
	\leftarrow \rangle_x				
	\rightarrow $\rangle_x, \leftarrow \rangle_x$				