AN ABSTRACT OF THE THESIS OF

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Abstract approved: ______________________________

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One source of difficulty for students in Thermodynamics courses is their unfamiliarity with partial derivatives and the associated mathematical procedures. At Oregon State University, a mechanical analogue of thermodynamic systems, called the Partial Derivative Machine (PDM), has been designed. The PDM represents an attempt to make the mathematics of partial derivatives more accessible to students by not simultaneously introducing new physical concepts from thermodynamics, such as Entropy. The Interlude session associated with the upper-division thermodynamics course at Oregon State University introduces the mathematical techniques using this new learning tool, the Partial Derivative Machine. Using the Actor Oriented Transfer perspective this research examines the transfer of skills and knowledge developed with the Partial Derivative Machine in this intensive mathematical session to new physical contexts in the Energy and Entropy course and new physical contexts. Students from the course were recorded both in class and in recorded problem solving sessions to observe instances of transfer.

Key Words: Actor Oriented Transfer, thermodynamics, partial derivatives

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Examining Upper-Division Thermodynamics Using the Actor Oriented Transfer Framework

by

Grant J. Sherer

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Toni Doolen, Dean, University Honors College

I understand that my project will become part of the permanent collection of Oregon State University, University Honors College. My signature below authorizes release of my project to any reader upon request.

Grant J. Sherer, Author

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I. OVERVIEW AND GOALS

This project is part of the ongoing Paradigms in Physics Project, a complete redesign of the upper-division physics major at Oregon State University. This project focuses on the course content of the Energy and Entropy Paradigm and the Interlude session, an intensive week of mathematical methods, that precedes it. The current reformulation of this course is focused on providing students with more concrete examples and highlighting the physical significance of mathematical manipulations in thermodynamics problems through the use of new classroom activities and sequences of content. The primary goal of this Interlude is to develop mathematical relationships present in thermodynamics in a familiar mechanics context before beginning instruction in the more abstract subject of thermodynamics.

Students often struggle with thermodynamics at least in part due to the unfamiliarity and complexity of partial derivatives [1–4]. Research conducted to assess student applications of partial derivatives in thermodynamics has shown that even when given tasks related to abstract non-physical contexts students struggle to correctly express and use partial derivatives [5, 6]. During the 2012-2013 academic year, we designed an apparatus, which we have called a Partial Derivative Machine (PDM), with the goal of introducing partial derivatives in a physical context that is familiar to students through a mechanical analogue of a thermodynamic system. This apparatus, along with the activities described in this paper, represents a small piece of an extensive redesign of Energy and Entropy, which is the thermodynamics portion of the Paradigms in Physics sequence at Oregon State University [7]. This reform primarily relies on development of Interlude materials that build understanding of partial derivatives through more concrete and tangible applications of derivatives. The PDM attempts to meet our goal of connecting partial derivatives, tangible physical systems, and more abstract thermodynamic systems. By incorporating the PDM into Interlude activities which are mathematically analogous to the Energy and Entropy course material, students are able to study a physical system which can be manipulated tangibly.

Using the Actor Oriented Transfer framework to study students solving assignments where a thermodynamics problem has been paired with a similar mechanics problem earlier in the term we plan to investigate the following research question: Do students make explicit references to connections they see between the task at hand and exercises using the PDM, make gestures indicative of working with the PDM, or repeatedly use mathematical techniques?

II. UNDERLYING PRINCIPLES: HOW THE PDM WORKS

The Partial Derivative Machine is an apparatus consisting of a central spring system that can be stretched via two strings extending outward from the center (See Fig. 1). The thumbnuts in the center of the board provide a fixed point to keep the spring system mounted to the board as masses are added. This central system is placed on a large piece of particle board which features a pulley and a knob on each of two adjacent corners (A and B in Fig. 1). Fixing the string with the knob at A or B allows the user to fix the respective position (x or y) while performing other manipulations on the system.

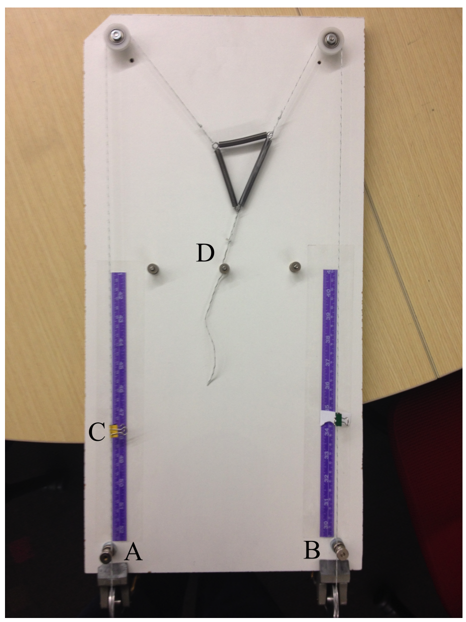


FIG. 1. The PDM, (A/B)Pulley and knob, (C)Measuring Flags, (D)Mounting Knobs.

In order to more easily measure the stretching of the system, a measuring tape is placed on the board parallel to each string and flags are added to the strings (Example labeled C in Fig. 1). By defining the “X” and “Y” axes, the instructor is able to define four quantities for this experiment: x, the position of the X flag; y, the position of the Y flag; F_x, the tension in the X oriented string; F_y, the tension in the Y oriented string.

FIG. 2. Pulley and Knob (A in Fig. 1).

There are two methods that can be used to manipulate the system. The first method involves tightening the knob on corner A or B to pin the corresponding string, thereby fixing x or y, and then increasing the mass on the opposite (freely hanging) string. For example, pinning the knob at B would fix y, then adding weight to the X string would increase x, F_x, and F_y.

Alternatively, one can leave both the X and Y strings free and add weights to one or both. In doing so, placing weight on a string causes the system to react and both x and y change to allow the system to reach a stable state. For example, adding weight to the X string would cause x and F_x to increase while F_y stays constant and y may increase or decrease.

A. Example Central Systems

A variety of central systems can be used with the PDM to provide students with different experiences. The variety of spring configurations leads to different mathematical and physical behaviors of the system. During the first activities with the PDM, these central systems are hidden from students with a black box as in Fig. 4 in an attempt to make it impossible for students to use Hooke's law for their system. Four example systems are pictured in Fig. 3 to illustrate the different ways that the two strings can be coupled. The systems included in Fig. 3 each behave slightly differently and consequently each group finds different relationships to describe the behavior of the system. Not included in this picture are two systems only used for the first activity "Simple Derivatives" described in Section VI B. The first of these systems is a single spring connected to each string with no coupling; this system behaves uniquely because manipulation of the X string has no influence on the Y string and vice versa. The second system contains no springs, only strings, and it is therefore possible for the system to be maximally stretched in particular direction and no longer respond to addition of additional force in that direction.

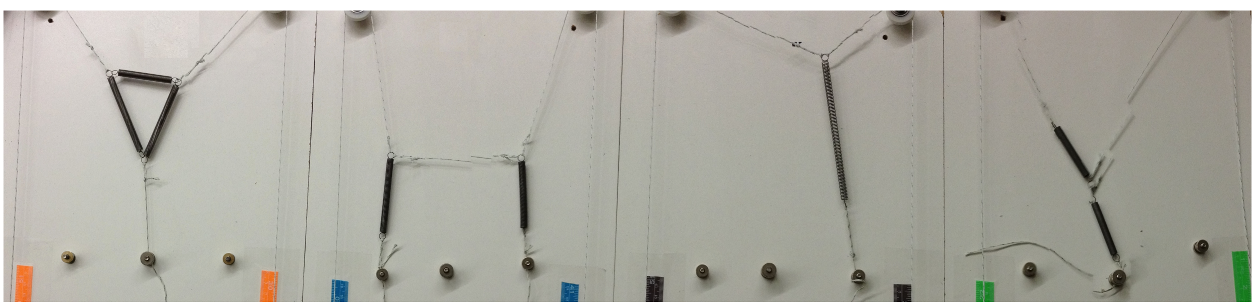


FIG. 3. Example "central systems" used with the Partial Derivatives Machine

B. Changes from 2013 to 2014

The previous version of the Partial Derivative Machine used in the 2013 session of Interlude (detailed description in Appendix C) featured four strings extending from the central system. Two of these strings only functioned to hold the system in place and made experimentation more difficult as they required frequently re-centering. The definitions of F_x and

FIG. 4. PDM with “Black Box” included.

F_y used in the 2013 course required the two forces to be orthogonal, a restriction that was not satisfied if the system was off-center. Additionally, if the system was not re-centered to account for the change in forces it was prone to slip off the pulleys. The knobs on the new system (D in Fig. 1) have replaced the two strings used to position the central system in the 2013 version.

The new system has the benefit of a layout of pulleys that minimizes occurrences of the strings slipping off as well as only requiring two strings to manipulate the system. The fact that the system can only be manipulated with two strings attempts to clarify to students that there are exactly two ways of putting energy into the system (one per string). The previous model created confusion as to whether or not recentering the system with the third and fourth strings required putting energy into the system. A significant benefit of the new system is that the measurements students took could be done in shorter time since no time was spent repositioning the system. Since the time to conduct a measurement was reduced it was the goal of the instructor to present more opportunities for measurement into the course.

III. LITERATURE REVIEW

The ability to apply knowledge from one context to another is often defined in the Education Research community as *transfer*. Transfer has its foundations in both education and psychology research and has evolved into many sub-frameworks each with their own definition of what constitutes evidence of transfer and how it should be assessed. In traditional models of transfer, transfer is treated as the ability of students to apply content learned in one setting to identical problems in a new setting. In more recent approaches the framework has expanded to view transfer as a dynamic process that treats the target situation, the situation in which the researcher looks for transfer, as a potential learning situation. Additionally, modern perspectives reconsider whether it is the researcher or subject who should identify the initial and target situations as similar.

One contemporary framework by Bransford and Schwartz views transfer as “preparation for future learning” (PFL) focusing on problem solving ability in new contexts [8]. This approach analyzes the processes learners use to solve problems and whether the subject can learn to use methods developed in previous contexts in the target situation. In this approach, the subject is not “sequestered” but is instead allowed to use additional resources, revise their thinking, and receive feedback . The developers of the PFL framework consider the “sequestered problem solving” and “direct application” as a possible explanation for the perceived pessimism surrounding evidence of transfer. Instead, the authors promote an examination of how initial learning situations foster an ability to learn in new situations. The example provided by the authors is a study of subjects’ ability to learn a new text editor given previous experience with text editors. From a PFL perspective, examination of potential transfer scenarios involves direct exploration of people’s ability to learn and relate new information. This perspective shifts the focus to whether learners are prepared to learn to solve new problems. In the case of the text editor task, examination of these scenarios would study the questions learners ask because these provide a partial picture of their learning goals.

Wagner's Transfer in Pieces approach considers the development of knowledge across several encounters with the same task [9]. In this approach, transfer is evident through the transition from initially perceiving a set of problems as different to developing a conceptual understanding of how they are "alike" in terms of underlying principles. A critical element of Transfer in Pieces, and the resource framework as a whole, is that coordinating ideas across multiple situations is an essential characteristic of some concepts themselves. Consequently, Transfer in Pieces does not make a firm distinction between general understanding of a concept and the ability to apply it in specific situations. Transfer in Pieces draws from the idea of resources in learning and traces these knowledge resources across the sequence of problems and studies how these ideas develop. From this analysis, transfer is viewed as "the incremental growth, systematization and organization of knowledge resources that only gradually extend the span of situations in which a concept is perceived as applicable".

Another contemporary approach is Lobato's Actor Oriented Transfer which focuses on the learner and their interpretation of situations as similar [10]. Actor Oriented Transfer (AOT) focuses on instances in which the research subject is able to make a connection between contexts, through "personal construction of relations of similarity across activities". The framework has two foci, the learner and the learner's perception of the initial and target learning situations [12]. Evidence of AOT is the existence of student-identified relationships between the initial learning situation and the task at hand observed in one of two ways.

Specifically, any instance where the learner makes an explicit reference to previous learning situations is considered evidence [11, 12]. The second type of evidence is less obvious and occurs when the learner makes repeated use of particular techniques often without articulating that they are making a connection. Consequently, the judgment of the researcher impacts the consideration of these situations as evidence of transfer. While traditional cognitive views of transfer identify the ability to apply formulas and surface features (procedural steps) across multiple problems as evidence for transfer, AOT examines student reasoning processes since students could be making connections from initial learning situations the researcher does not expect [12]. A combination of think-aloud problem solving and analysis of the learners' procedure can be used to identify their reasoning on a particular task.

IV. METHODS

A. Choice of AOT Framework

AOT was used as a theoretical framework because we wanted to determine whether students were able to apply conceptual understanding gained from the PDM to thermodynamic topics. Alternative transfer frameworks were considered for this project, but the "evidence of transfer" in other frameworks examines students' ability to perform a particular task. In the context of this project, this evidence would be measured by students demonstrating increased ability to solve mathematically identical problems after completing the problem in the Interlude and Energy and Entropy with different variables. Since it is the purpose of the Interlude to develop these skills the researcher felt it would aid in curriculum development to investigate the transfer of conceptual understanding and identify instances where students noticed similarities between problems. In previous years the Interlude has taught the mathematical ideas without any physical context, with abstract variables, and with the variables (but not physics) of thermodynamics. A benefit of the PDM may be the opportunity to provide physical significance and conceptual understanding of the variables, derivatives, derivative relationships, and total differentials used in Interlude. Consequently, AOT might provide an appropriate framework for analysis of the Interlude and Energy and Entropy courses due to its focus on transfer of concepts in addition to the procedural steps required for problem solving. In our study, AOT would be evident through explicit references, such as those by name or conceptual connections, and gestures indicative of working with the PDM.

B. Participant Selection

The original plan for participant selection for this study was to recruit volunteers via an email sent to a particular subset of the students in the course, see appendix A. One selection criterion was student attendance in previous courses. Choosing students with reliable attendance habits was done to increase the likelihood of reliable subject involvement. An attempt was also made to select students, and subsequently form groups, at varying ability levels as demonstrated in previous Paradigms in Physics courses. The ability of students to explain their reasoning while solving problems was also considered as a factor in selection of potential volunteers. This selection was done by the course instructors and the researcher was not informed of the justification for the selection of any individuals.

Due to limited response to this initial invitation, Professor Manogue appealed to the PH461 Mathematical Methods course, which most Paradigms students are concurrently enrolled in, for additional participants. After this appeal, five additional volunteers were found for a total of six. These students were then divided into two groups for the duration of the course. Groups were formed based partially on schedule availability but also took into consideration the creation of groups that would be comfortable working together. Professors Manogue and Roundy also considered the grades of the students to create groups with a range of abilities. Each group was assigned to a particular workspace in the classroom to aid in data collection as described below.

C. Data Collection

Data collected during this project consisted of recorded problem solving sessions and classroom video from the 2014 Energy and Entropy Paradigm. All data collected was stored on a password protected network attached server (NAS) in the Weniger 495 lab facility.

Problem solving sessions were conducted in the Paradigms Research Group's lab space, Weniger 495. One camera was placed overhead to record the work of students on large white boards as they completed tasks. A second camera recorded from the corner of the room to record students gestures, facial expressions, interactions, etc. Sessions consisted of students doing group problem solving exercises that presented problems included in the homework assignments for Energy and Entropy, but before the students had been asked to submit the problems for a grade. Students were asked to converse with each other while working on tasks, articulate the steps they were taking, and explain the motivation for doing so. During these problem solving sessions, students were allowed to work with little input from the interviewer. Occasionally, clarifying questions were asked of the students so the interviewer could follow the student work (such as asking for axis labels on a graph) but the interviewer neither assisted students in solving problems nor asked them to explain their reasoning on a particular task. Students were allowed to consult the instructor or teaching assistants as needed so that the participation in the problem solving session would not impede their ability to make progress on the assignment. During several problem solving sessions the students were clearly not progressing and the researcher asked teaching assistants or instructor to join the session and to provide help as needed.

Classroom video was collected during each class period during Interlude and Energy and Entropy, a total of four weeks. Two cameras were focused on each of the groups involved in this study to record their work. The first was positioned above the group and was used to record the work of students on large white boards and working with the Partial Derivative Machine. The second camera was positioned across the table from the group and recorded conversations between students and provided a better viewing angle to observe facial expressions and interaction within the group. A fifth camera and microphone were used to record the instructor during lectures and group activities. This provided a comprehensive record of what the instructor said, whether different groups received different instruction, and specifically what was presented to the research participants.

D. Problem Selection

One task from each homework assignment was selected for use in each problem solving session. During the first and second sessions, students worked to complete their analysis of data taken from laboratory experiments done in class. First was the Potential Energy Lab from Interlude used with the PDM (See Homework 1 in Appendix B). This lab was selected primarily because data collection had occurred the same day as the interview session and the assignment was due the following day. Second was the Ice Calorimetry Lab from Energy and Entropy (Homework 3 Appendix B). This task was also selected for the problem solving session due to the timing of data collection and the collection of the assignment. These two assignments represented a larger amount of work than a traditional problem set so it was decided that students should be able to work on the assignments before the group session and use the group session time to continue their work.

The third session focused on question 5.3 from the fifth homework set (See Homework 5 in Appendix B). Our research group selected this problem for use during the interview session because we thought this task might present a likely scenario for transfer. For this session, students were asked to wait to begin work on the problem until the interview session so that we could observe them completing as much of the task as possible. This problem asked students to use the analytic expressions of tension and heat capacity of a bungee cord to find analytic expressions for the free energy, internal energy, and entropy. Aside from the obvious analogy of something being stretched this problem was unlike any particular task done in Interlude as the concepts required for this problem were not identical to those needed for problems done with the PDM.

Problem 5.3: Bungee A physics major carefully measures the tension in a Bungee cord over a range of temperatures from room temperature to the boiling point of water. She examines her data carefully and finds that the tension in the cord is very well approximated by

$$\tau = \left(a - be^{-T/T_1}\right) \tan\left(\frac{\pi L}{2L_M}\right)$$

where L is the length that the cord is stretched beyond its relaxed length, and a, b, T_1 and L_M are positive constants.

She then places the relaxed cord ($L = 0$) in a calorimeter and measures the heat capacity over the same range of temperatures and finds that

$$C_L = \alpha + \gamma e^{-T/T_1}$$

where α and γ are two additional positive constants, and T_1 is the same value found in the previous experiment.

a) Sketch the tension τ versus the stretch L, and the heat capacity C_L versus the temperature T.

b) Find the change in free energy $\Delta F = F(T, L) - F(T_R, \frac{1}{2}L_M)$ where T_R is room temperature.

c) Solve for the change in entropy $S(T,L) - S(T, \frac{1}{2}L_M)$ at an arbitrary temperature and length.

d) Solve for the change in internal energy $U(T,L) - U(T, \frac{1}{2}L_M)$ at an arbitrary temperature and length.

Problem 5.3, the task used in the third problem solving session

The final session used question 7.2 which was almost identical to question 2.3 which was completed on a previous assignment. The full assignments can be found in Appendix B. These questions required use of the ordinary and cyclic chain rules to show the equality of two quotients of derivatives. The only difference between the two problems was the set of physical variables (S, T, p, V vs. x, y, F_x, F_y) expressed in the derivatives. With this task, we thought there might be opportunities for transfer of conceptual ideas about what the quotients represented in addition to the students ability to solve the same mathematical problem in a new context.

Problem 2.3: Isowidth and isoforce stretchability In class on Tuesday, you measured the isowidth (or Corinne) stretchability and the isoforce stretchability of your systems in the black boxes. We found that for some systems these were very different, while for others they were identical.

Show that the ratio of isowidth stretchability to isoforce stretchability is the same for both directions of a given system, i.e.:

$$\frac{\left(\frac{\partial x}{\partial F_x}\right)_y}{\left(\frac{\partial x}{\partial F_x}\right)_{F_y}} = \frac{\left(\frac{\partial y}{\partial F_y}\right)_x}{\left(\frac{\partial y}{\partial F_y}\right)_{F_x}}$$

Problem 2.3, taken from the second homework assignment

Problem 7.2: Isothermal and adiabatic compressibility The isothermal compressibility is defined as $K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$. K_T is found by measuring the fractional change in volume when the pressure is slightly changed with the temperature held constant. In contrast, the adiabatic compressibility is defined as $K_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S$ and is measured by making a slight change in pressure without allowing any heat transfer. This is the compressibility, for instance, that would directly affect the speed of sound. Show that

$$\frac{K_T}{K_S} = \frac{C_p}{C_V}$$

Where the heat capacities at constant pressure and volume are given by

$$C_p = T\left(\frac{\partial S}{\partial T}\right)_p \text{ and } C_p = T\left(\frac{\partial S}{\partial T}\right)_V$$

Problem 7.2, the task used in the fourth problem solving session

V. INSTRUCTOR GOALS FOR THE PARTIAL DERIVATIVE MACHINE

Before the 2014 Interlude course, Professor Corinne Manogue and Professor David Roundy were each interviewed by the researcher to establish what they felt was valuable about using the PDM. These interviews were conducted separately in the Paradigms Research Group lab space during Winter Term 2014. The interview started with a discussion of the interviewees own difficulties with thermodynamics both as a student and an instructor. From this, each interview moved towards a conversation about the difficulties the instructors perceive students having in thermodynamics. These ideas built up to a conversation with each instructor of their own learning goals related to the use of the PDM in the Interlude. A summary of each instructor's learning goals is given below.

A. Professor Corinne Manogue

Professor Manogue expressed a number of learning goals related to the use of the PDM in the Interlude course. One common student difficulty is a struggle with interpreting the degrees of freedom in a thermodynamic system. The tangible nature of the Partial Derivative Machine is useful for helping students identify how many free parameters are available through actual manipulation of a system.

It is also a goal of Prof. Manogue that the Partial Derivative Machine be used to help students understand conjugate variables and the relationship between properties of the system. This topic is explored when dealing with the PDM equivalent of the First Law of Thermodynamics and builds towards an understanding of internal energy as a function of the state of a system. This is an important topic for students to develop a sense of what "conservative" means from a physical perspective besides the mathematical viewpoint of "a curl is zero” or “a potential exists”.

The PDM also provides students an observable system to build understanding of the mathematical and physical consequences of the variable(s) held fixed when taking derivatives. Prof. Manogue emphasized that the PDM is valuable in demonstrating “derivatives as experiments”, a complement to the “Name the Experiment” activity sequence in Energy and Entropy, that helps students understand what is happening when a particular derivative is measured.

Introducing Legendre Transforms and “free energies” with the PDM provides the instructors an opportunity to develop student understanding of the physical significance of these manipulations. The physical significance of Legendre Transforms can be lost in the mathematics in a thermodynamics context, but with the PDM the instructor can explore these transformations as if it is including one of the hanging weights “inside the black box” representing internal energy.

B. Professor David Roundy

Professor Roundy listed concepts and ideas that he felt were useful to explore with the PDM. The first of these topics was the various properties of derivatives that can be explored through the hands-on nature of the PDM. Demonstrating that derivatives are invertable with the PDM is useful because, while it may seem mathematically legitimate, the PDM allows students to demonstrate this property with measurement. It is also possible for students to use measurement to observe that the variable held fixed matters when taking a derivative. Prof. Roundy also hoped to use the activities and measurements performed with the PDM to help students learn two properties of derivatives. First, derivatives are functions of system variables such as x and y. Second, derivatives represent a variable themselves and are consequently not always constant. Using the PDM hopefully aids students in developing an understanding of derivatives as an experiment through a combination of mathematical computation and physical manipulation.

Asking students to physically measure derivatives with the PDM may help them understand both the process of obtaining Maxwell Relations from expressions of the Energy and to verify the accuracy of the Maxwell Relation. These concepts connect to student understanding and manipulation of total differentials that are used extensively in thermodynamics. It is important for students to understand total differentials as a relationship between small quantities and interpret them mathematically.

Finally, Prof. Roundy discussed the importance of developing understanding of the "state" of a system. It is important that students realize that for a thermodynamic system, or the mechanical equivalent presented by the PDM, the state of a system defines all the variables (x, y, F_x, F_y or T, S, p, V), but it is not necessary to use all four variables to define the state. This leads to the idea that students should understand the relation between properties of the system and be able to count the number of properties that can be used to influence the system.

VI. INTERLUDE CLASSROOM SEQUENCE, 2014

A. Day 1: Introduction

When first introduced to the Partial Derivative Machine, the central system was hidden from students through the use of a "Black Box" (See Fig. 4). With only the knowledge that there were two strings extending from this box, students were told that there were two measurable properties (typically x and y) and two controllable properties (typically F_x and F_y).

Students were then asked to determine how many properties could be controlled independently. Students worked briefly in groups of three to answer this prompt and then were brought back together for a class discussion.

Many students did not realize that the tension in a particular string is not equivalent to the weight hung from that string if the corresponding knob is locked since the mass becomes irrelevant when the string is pinned down. Most students determined two of these properties could be controlled independently, and that manipulating a pair of parameters caused a responsive change in the other parameters but had difficulty coming to the conclusion that the user is able to determine which properties were independent in a given scenario.

B. Days 1 and 2: Finding a Simple Derivative

Once students were familiar with the machine, they were asked in a second exercise to measure $\frac{\partial x}{\partial F_x}$ and provide a numerical answer with units. In this exercise students might notice that there were two possible options: $\left(\frac{\partial x}{\partial F_x}\right)_y$ and $\left(\frac{\partial x}{\partial F_x}\right)_{F_y}$. Roughly one half of the class chose to measure each derivative; some groups noted that they were holding a particular variable constant while others did not. The instructor used this as an opportunity to define the concept of stretchability as it relates to the system and distinguished between the "isoposition" (constant y) and "isoforce" (constant F_y) stretchabilities.

After collecting data sets, some students approximated one or both derivative(s) with a difference quotient while others used an intermediate step of plotting the derivative. The approach of some groups was to take a few measurements of the form (F_x, x) and approximate the derivative with the quantity $\frac{\Delta x}{\Delta F_x}$. Other groups chose instead to plot x as a function of F_x and either make linear approximations of the function or take derivatives of the best fit to find a numerical value. Each method led students to numerical values of the derivative. The groups then reported these values to the instructor and briefly discussed their data collection methods.

Due to the different systems under the black boxes, the numerical values for the "isoposition" stretchability and "isoforce" stretchability varied widely from group to group. The configuration of the springs impacted many behaviors of the system including the coupling between the two strings and the rigidity of the system. The original state of the system when the measurements were taken also impacted the numerical value students found as systems behaved differently when stretched. The relationship between x and F_x also varied from system to system due to the configurations of strings and springs used.

C. Days 1 and 2: Integrated Lab

As a preface to a major activity associated with the Partial Derivative Machines, students were given a review lecture (Day 1) on:

- Calculating changes in internal energy, ΔU, as the work, W, done on the system

- Work as the integral of force, $W = \int F dx$
- The number of energy inputs (two) for this system, since each string is one way to do work on the system.

Students then worked for the remainder of the period on determining a method of data collection for the lab experiment. Students did so in the same groups as in previous activities and continued to work in these groups for the remainder of the course. The development of a data collection method was the first homework task assigned to the class and was treated as a pre-lab task for the Potential Energy Lab.

Problem 1.1: Planning Ahead Work in groups to answer the following prompts. Bring one copy of your groups plan to class Tuesday.

a) What data will your group take?

1. Which weights will you use?
2. What x or y constraints will you use (if any)?

b) How will you organize your data? (table, graph, etc.)

1. If you use a table, include a copy of your table when turning in your plan. Make sure your table reflects the plan you outlined in the previous question (ex: include values of independent variables where appropriate).
2. If you use a graph, turn in a template with axes and include a sketch of what you expect the data to look like.

c) How will you analyze the data you collect?

Before you arrive in class on Tuesday to take data, you should draft a plan of attack for collecting data. In particular, you should keep in mind that you will need to find the potential energy as a function of two orthogonal coordinates. Since the work only gives you a *difference* in potential energy, you need to ensure that you have sufficient data in order to find the potential energy at every point you consider relative to the same arbitrary zero.

On the second day of the course, after reviewing and completing the "Simple Derivative" activity, students proceeded to conduct a laboratory experiment. The primary task was to measure the potential energy stored in the spring system of the Partial Derivative Machine, however, a process to determine the energy of a particular state was not explicitly given. The review of work, potential energy, and springs prior to data collection was designed to help students make the connection that the potential energy could be obtained from the work done on the system. Since the system was two dimensional, using $W = \int F dx$ required finding the work done on the system in both the X and Y directions.

One possible solution method that determines all necessary information is:

1. Starting at a particular $x = x_o$, where $\Delta x = 0$, take measurements of y while changing F_y (by adding/subtracting masses) in uniform steps, *e.g.,* $0.05kg$.

2. Set subsequent x values by loosening knob B, incrementing F_x by small uniform steps, and then tightening knob B.

3. Repeat step 1 for each new fixed x value.

4. Using the data and numerical integration of $F_x\, dx$ and $F_y\, dy$, approximate the value

of $U(x, y)$.

This process gave students the data needed to get from any state (x_1, y_1) to a different state (x_2, y_2), provided each corresponded to a state generated during the steps outlined above. To verify path independence one would need to conduct a similar process, now measuring x for fixed y values while varying F_x (changing F_x and F_y by the same increments used above). This lab also provided students practice distinguishing between fixed y and fixed F_y processes and the relevance of each to particular measurements.

One common difficulty that students had when collecting data and performing their analyses was a lack of sufficient data to complete the calculations. The data collection and analysis methods many students used allowed them to find the change in internal energy along any fixed x or y line but did not allow them find the energy along a combination of these paths. Students are able to calculate the change in internal energy along a combination of fixed x and fixed y lines easily if the intersection of the two lines corresponds to a state (x, y) at which they collected data. If the intersection was a point not in their data set the calculation of ΔU required interpolation, an option many students missed.

D. Connections to Thermodynamics

The integrated lab was designed to allow students to reach the conclusion that there are two ways to manipulate the potential energy of the system, each corresponding to a particular force (F_x or F_y) and distance (x or y). The students were able to see the pairing of these quantities through the differential expression for work,

$$dU = F_x dx + F_y dy \tag{1}$$

which is a mechanical equivalent of the thermodynamic identity, using conjugate pairs of forces and distances rather than the canonical thermodynamic conjugate pairs: temperature and entropy, pressure and volume.

The instructor then introduced the concept of the potential energy of the system, U, as a state function determined by x and y, an implicit function whose total differential is given by:

$$dU = \left(\frac{\partial U}{\partial x}\right)_y dx + \left(\frac{\partial U}{\partial y}\right)_x dy \tag{2}$$

From Eqs. 1 and 2 the instructor extracted definitions for the forces as partial derivatives of the potential energy, a result which should be familiar from both classical mechanics and Electricity and Magnetism.

Using these expressions for F_x and F_y students were asked to express $\left(\frac{\partial F_x}{\partial y}\right)_x$ and $\left(\frac{\partial F_y}{\partial x}\right)_y$ as derivatives of U. From these new derivative expressions, and Clairaut's theorem, which states the order in which mixed partial derivatives are taken does not matter, students found a Maxwell Relation for their system that they could experimentally verify.

E. Days 3-5: Additional Mathematics Content

During the remaining contact hours of the Interlude course, using the physical parameters of the PDM, the instructor led students through additional mathematical techniques relevant to thermodynamics including taking reciprocals of partial derivatives, other partial derivative manipulations, the cyclic chain rule, and Legendre Transforms. These exercises allowed students to practice these math methods and understand their results in the context of tangible experimental measurements. Describing Legendre Transforms as "extending the black box to include one or more strings" attempted to help students understand the conceptual and mathematical value of this technique.

The instructor allowed students to conduct measurements of "simple derivatives" and manipulations in order to periodically verify the expressions determined for change of variables and partial derivative chain rules. These brief exercises were used to break up lecture and provide students additional hands-on learning exercises.

The instructor also used the variables of the PDM and the associated mathematics as a means of teaching about total differentials and "the three things you can do with total differentials". These mathematical manipulations are algebraic manipulation, interpretation of coefficients as derivatives, and integration along a path.

VII. ANALYSIS

A. Evidence of Actor Oriented Transfer

During the second problem solving session the groups were asked to work on the Ice Calorimetry Lab (Homework 3) having collected the data the previous day in class. While planning their approach to the final question on this assignment one student remarked "this is like what we did last week" referring to the numerical integration techniques to find the change in Entropy in Question 3.5 as similar to those used to find Internal Energy in the Potential Energy Lab (Homework 1). We view this event as evidence of AOT as it is clear from a combination of the context of the quote and the quote itself that the student has made a connection between these two instances where numeric integration of their data provided a correct solution. The student in this example was exhibiting transfer of a procedural skill developed in the Interlude but this incident does not provide strong evidence of transfer of conceptual understanding between the PDM and thermodynamic systems.

In the final problem solving session, students were asked to complete a task (Problem 7.2) that was mathematically identical to a task from a previous assignment (Problem 2.3). When solving this task, both of the participant groups made attempts at solution methods that differed from the original solution. One possible explanation of this is that in the first exposure to the problem the class was provided a hint that the ordinary and cyclic chain rules for partial derivatives may be useful. When exposed to the problem a second time, the students had much more experience with a variety of mathematical techniques that are often useful in solving thermodynamics problems but without the hint were unsure of which step would work best in this context.

No other explicit references were made to the Partial Derivative Machine or techniques used in the Interlude during the problem solving sessions, but this does not imply that students did not learn from the PDM or think about it when solving problems. Limited opportunities for the observation of AOT were provided by only examining the conversations between students and their explicitly stated reasoning. It is possible that students were able to construct similarities between activities or topics in Interlude and Energy and Entropy but did not verbalize them. It is also a possibility that the problems were so different that the analogies the instructor hoped to develop were not recognized as easily by the students.

A more comprehensive examination of transfer in this course could be accomplished by designing and implementing paired assignments and classroom activities that require identical mathematical procedures or better parallel the conceptual ideas used in Interlude. A consequence of this change might be a shift of the focus away from conceptual similarities across contexts and towards procedural fluency with the mathematics.

B. Reflection on Problem Selection

The problems used for the problem solving sessions may have contributed to the lack of evidence of Actor Oriented Transfer observed. As discussed above, it may be the case that the concepts covered on the homework assignments during the Energy and Entropy Paradigm were so conceptually different from the tasks completed in the Interlude that transfer may have been unlikely. Questions 7.2 and 2.3 were nearly mathematically identical yet the students did not make explicit connections between the two when solving them during problem solving sessions. Other pairs of problems were not as closely aligned conceptually or mathematically. For example, the third problem solving session used question 5.3 which relied on integration of analytic forms of expressions whereas Interlude activities were focused on numerical integration.

C. Instructor References to Partial Derivative Machine in Lecture

There were multiple occasions during the Energy and Entropy Paradigm that Professor Roundy made use of a PDM at the front of the class to illustrate particular concepts. One such situation was during the first class period when Professor Roundy discussed transferring energy into and out of a system in the context of the Ice Calorimetry Lab:

> *Most of these (stirring the cup of ice water, irradiating the cup, putting the cup in contact with a hot object, etc.) work basically by heating. They heat the system which relates to sort of a random transfer of energy that does nothing but heat the system up. The other approach you could do is compress [the water]... These two ways to transfer energy are the thermal analogues of the two strings in the Partial Derivative Machine. If you throw in chemistry, you get more strings. You get more ways you can transfer energy into and out of a system if you allow chemical reactions.*

Another instance in which Professor Roundy referenced the PDM was when he explained Enthalpy and Legendre Transforms in the context of the Ice Labs, drawing connections between the equivalent physical and mathematical manipulations in Interlude:

> *We talked about this last week when we used the Partial Derivative machine and I talked about what if you were to cover up one of those weights. If you cover up one of those weights and you can't measure the position of that weight then you*

can't find out what the potential energy of the system is but you can still measure the work done [on the other string]. That is exactly like your experiments with Styrofoam cup you couldn't measure the volume conveniently so you don't know what this [$-pdV$] term is. We introduced with the machines the idea of taking the second energy from the second work and adding it on in a sort of interesting manner.

A third instance in which Professor Roundy made reference to the PDM during lecture was to emphasize the importance of the measurement aspect of thermodynamics and how derivatives can be interpreted as measurement. One example of this was at the start of the Energy and Entropy Paradigm when students were asked to respond to a prompt to define temperature:

What I was surprised that no one wrote was something like temperature is the thing you measure with a thermometer. That is actually really fundamental, that we can define what it is in complicated math terms, but it is really important that temperature is also the thing you measure in order for us to understand what it is. Just like when we talked about forces last week it was important to say force is the derivative of the potential energy with respect to x, but is also the thing you measure by measuring how much mass there is.

In each of these instances, Professor Roundy was attempting to make conceptual links between the Partial Derivative Machine and thermodynamic systems. The third instance is more abstract in considering measurement and temperature than the first and second which each made explicit references to experiments the students performed but each of these excerpts demonstrates an attempt to make a thermodynamic idea more tangible through references to the PDM.

D. Ideas for Future Use of Partial Derivative Machine

One possible redesign of the Energy and Entropy Paradigm could make use of the Partial Derivative Machine during the Paradigm instead of a devoted week of separate content. Addressing the same mathematics content in two separate physical contexts separated by a week may impact the development of conceptual links between the PDM and thermodynamics. If instead the instructor incorporated the activities using the PDM into the Paradigm and blended its use with the other lab assignments and classroom activities used in the thermodynamics portion of the course perhaps the conceptual links and mathematical skills could each be developed more. The time separation of "paired" activities using similar mathematics in the two contexts may have contributed to the lack of observed Actor Oriented Transfer and decreasing this separation could possibly benefit students by placing greater emphasis on the conceptual similarities to complement the mathematical parallels.

VIII. SUMMARY

Using the Actor Oriented Transfer (AOT) framework, this research project examined the Interlude and Energy and Entropy courses at Oregon State University for evidence of student application of conceptual and mathematical ideas between the courses and the physical contexts each presents. In Interlude, students are exposed to a mechanical system, called the Partial Derivative Machine (PDM), designed to build conceptual understanding of thermodynamics and fluency with the mathematical procedures needed for thermodynamics. Problem solving interviews were conducted during the Energy and Entropy Paradigm to observe whether or not students made explicit references to the PDM when completing homework tasks, made gestures indicative of working with the PDM, or applied similar mathematical reasoning between the two contexts.

During the second problem solving interview one student drew parallels between the use of numerical integration techniques in both the Ice Calorimetry Lab (Homework 3) and the Potential Energy Lab (Homework 1) completed in the Interlude. We view this event as evidence of AOT as the student was able to apply mathematical and computational techniques from the mechanical system to thermodynamics and made explicit verbal reference to the previous assignment.

While only one instance of transfer was observed during the problem solving interviews, we do not view this as evidence that transfer did not occur. It is possible that the participants in the study made reference to the PDM and associated activities while doing group work and activities in class. The students' work in class was also recorded and may be analyzed in future research conducted by the Paradigms in Physics Research Group. Classroom video of the instructor was also collected and could be of use to future research projects.

One aspect of this project that may be modified for future research is narrowing the focus of the project. Our examination of AOT during the Interlude and Energy and Entropy did not have a specific focus on development of a particular skill, problem solving strategy, or conceptual understanding of a topic. Instead, in further research it would be beneficial for the researcher to examine the transfer of a particular idea such as understanding partial derivatives as measurements rather than examining whether students refer to activities from the Interlude while solving thermodynamics problems.

[1] D. Meltzer, PERC Proceedings PER Conference Invited Paper, **790**, 31 (2004).

[2] D. Meltzer, Physics Education Research Conference 2009 PER Conference Invited Paper, **1179**, 31 (2009).

[3] J. Thompson, C. Manogue, D. Roundy, and D. Mountcastle, PERC Proceedings PER Conference Invited Paper, **1413**, 85 (2011).

[4] D. Roundy, M. B. Kustusch, and C. Manogue, American Journal of Physics (2013), in press.

[5] J. Thompson, B. Bucy, and D. Mountcastle, PERC Proceedings PER Conference, **818**, 77 (2005).

[6] B. Bucy, J. Thompson, and D. Mountcastle, PERC Proceedings PER Conference, **883**, 157 (2006).

[7] "Paradigms in Physics: Course materials," physics.oregonstate.edu/portfolioswiki/courses:start (2013).

[8] J. Bransford and D. Schwartz, "Rethinking transfer: A simple proposal with multiple implications," (AMERICAN EDUCATIONAL RESEARCH ASSOCIATION, WASHINGTON, 1999) pp. 61–100.

[9] J. F. Wagner, Cognition and Instruction **24**, 1 (2006).

[10] J. Lobato and D. Siebert, The Journal of Mathematical Behavior **21**, 87 (2002).

[11] J. Lobato, Educational Psychologist **47**, 232 (2012).

[12] G. Karakok, "Students transfer of learning of eigenvalues and eigenvectors implementation of actor-oriented transfer framework," (2009).

[13] D. J. Ozimek, P. V. Engelhardt, A. G. Bennett, and N. S. Rebello, AIP Conference Proceedings

790, 173 (2005).

[14] L. Cui, N. S. Rebello, and A. G. Bennett, AIP Conference Proceedings **818**, 37 (2006).

[15] K. E. Gray and N. S. Rebello, AIP Conference Proceedings **790**, 157 (2005).

[16] G. Sherer, M. B. Kustusch, C. A. Manogue, and D. Roundy, PER Conference Proceedings (2013).

[17] C. A. Manogue, P. J. Siemens, J. Tate, K. Browne, M. L. Niess, and A. J. Wolfer, Am. J. Phys. **69**, 978 (2001).

[18] C. A. Manogue and K. S. Krane, Phys. Today **56**, 53 (2003).

[19] D. Roundy and M. Rogers, American Journal of Physics **81**, 20 (2013).

[20] C. A. Manogue, D. Roundy, M. B. Kustusch, E. van Zee, T. Dray, A. Gupta, and J. Wagner, PERC Proceedings (2013).

[21] J. F. Wagner, C. A. Manogue, and J. R. Thompson, AIP Conference Proceedings **1413**, 89 (2011).

Appendix A: Recruitment Email

Hello Energy and Entropy Students,

You are receiving this email because you have already agreed to participate in research in the Paradigms in Physics Program. As part of that research, we are seeking volunteers to participate in some extra research activities.

As a participant in this aspect of the study, you would be assigned to a specific group of 3 students for the duration of the Interlude (week 1 of Spring 2014) and the Energy and Entropy Paradigm (weeks 2-4 of Spring 2014). We would use video of your group as part of the research study. Additionally, on Tuesday afternoons during weeks 1-4 you would participate in videotaped problem solving interviews with your group in Weniger 495. The material you would work on during these interviews would be specific homework problems, not additional assignments. We request no more than one hour of your time for each interview and you are only asked to stay as long as it takes your group to complete the problem.

Please respond to Grant Sherer (only) by email whether or not you agree to be a part of this aspect of the study. There will be no negative consequences to you if you are unable or unwilling to agree. If you do agree, please let Grant know what hours between 2-8pm you would be available for the interviews (Tuesdays weeks 1-4 only).

We hope that our analysis of your work will lead to a better understanding of how to improve our courses here at OSU and at similar institutions nationally. This data will be analyzed by Grant Sherer for his undergraduate Honors Thesis this spring and also by subsequent students and researchers in the Paradigms Program. We all appreciate your willingness to consider being a part of this project.

Thank you,

Grant Sherer, Physics Undergraduate

Corinne Manogue, Director of the Paradigms Project

David Roundy, Assistant Professor of Physics

Appendix B: 2014 Homework Assignments

Homework 1
Potential Energy Lab

In this lab, you will take measurements on an elastic system that can be stretched in two orthogonal directions, which can control by applying two orthogonal forces. From these measurements, you must extract the potential energy of that system.

1.1 Work and potential energy

Changes in the potential energy can be found by computed by measuring how much work is required to get the system from one configuration to another. Keep in mind that the definition of work is

$$W = \int F_x dx + \int F_y dy$$

You will need to determine the right hand side of this equation carefully, measuring the force for changes in displacement. This will require *numeric* integration, which itself will require you to have a considerable amount of data.

Problem 1.1: Planning ahead Work in groups to answer the following prompts. Bring one copy of your groups plan to class Tuesday.

a) What data will your group take?
   1. Which weights will you use?
   2. What x or y constraints will you use (if any)?

b) How will you organize your data? (table, graph, etc.)
   1. If you use a table, include a copy of your table when turning in your plan. Make sure your table reflects the plan you outlined in the previous question (ex: include values of independent variables where appropriate).
   2. If you use a graph, turn in a template with axes and include a sketch of what you expect the data to look like.

c) How will you analyze the data you collect?

Before you arrive in class on Tuesday to take data, you should draft a plan of attack for collecting data. In particular, you should keep in mind that you will need to find the potential energy as a function of two orthogonal coordinates. Since the work only gives you a *difference* in potential energy, you need to ensure that you have sufficient data in order to find the potential energy at every point you consider relative to the same arbitrary zero.

1.2 Taking data

During class on Tuesday, you will take your actual data. Plan on spending only an hour taking data, as we will not devote the entire class period to data collection.

1.3 Bonus System (required for 523 students)

As a challenge problem (which is required for PH 523 students), you can take data—possibly outside of class—for a second system consisting of string with no springs, and analyze this data in the same way that you treat your primary elastic system. If you undertake this challenge problem, you should discuss in your report the differences and similarities between the two systems, and what makes them different.

1.4 Lab report

You will write up a formal lab report, which is due on Wednesday. This lab report should be written in good English, and should include all the standard parts of a lab report. In this report in particular, you should be sure to

- Include a diagram of your lab setup.
- Plot some of your raw data in a useful way.
- Plot U, being sure to account for the various dimensions present. It is not acceptable to hold any experimental parameter fixed for the entire analysis.

Please consult the rubric below for more information on the expectations for your lab report.

1.5 Rubric

Potential Energy Lab Report	Points
Abstract: *Abstract conveys full report effectively; briefly describes experiment, states goal and results.*	3
Introduction/motivation: *Gives clear reason or motivation for experiment. Is constructed to engage the reader.*	3
Experiment: *Describes apparatus, components and their relationship accurately, and provides clearly labeled diagrams. Describes procedure in sufficient detail that a peer could replicate.*	3
Results: *Physics is correct. Measured data is well-organized, described clearly, and plotted appropriately with attention to labels, captions, and presentation.*	6
Analysis: *Physics is correct. Quantities derived from raw measurement are clearly explained. Graphs and tables are clear and informative. Presentation addresses the multiple dimensions present in the problem.*	6
Assessment of data and conclusions: Compares experimental results with prior expectations. Provides an overview, a strong conclusion and comments on the experiment.	3
Language: *Language is clear, concise and descriptive. Spelling and grammar are correct. Tenses and voice are consistent. A professional tone is maintained. Organization is good.*	3
Title, date, acknowledgements, etc: *Title is a concise, informative description. Acknowledgements are appropriate. Date present. Pages are numbered.*	3
Total:	30

Homework 2

Problem 2.1: Rubber sheet Consider a hanging rectangular rubber sheet. We will consider there to be two ways to get energy into or out of this sheet: you can either stretch it vertically or horizontally. The distance of vertical stretch we will call y, and the distance of horizontal stretch we will call x.

If I pull the bottom down by a small distance Δy, with no horizontal force, what is the resulting change in width Δx? Express your answer in terms of partial derivatives of the potential energy $U(x, y)$.

Problem 2.2: Coffee and Bagels In economics, the term *utility* is roughly related to overall happiness. Many things affect your happiness, including the amount of money you have and the amount of coffee you drink. We cannot directly measure your happiness, but we *can* measure how much money you are willing to give up in order to obtain coffee or bagels. If we assume you choose wisely, we can thus determine that your happiness increases when you decrease your amount of money by that amount in exchange for increasing your coffee consumption. Thus money is a (poor) measure of happiness or utility.

Money is also a nice quantity because it is conserved—just like energy! You may gain or lose money, but you always do so by a transaction. (There are some exceptions to the conservation of money, but they involve either the Fed, counterfeiters, or destruction of cash money, and we will ignore those issues.)

In this problem, we will assume that you have bought all the coffee and bagels you want (and no more), so that your happiness has been maximized. Thus you are in equilibrium with the coffee shop. We will assume further that you remain in equilibrium with the coffee shop at all times, and that you can sell coffee and bagels back to the coffee shop at cost.[1]

Thus your savings S can be considered to be a function of your bagels B and coffee C. In this problem we will also discuss the prices P_B and P_C, which you may *not* assume are independent of B and C. It may help to imagine that you have

a) The prices of bagels and coffee P_B and P_C have derivative relationships between your savings and the quantity of coffee and bagels that you have. What are the units of these prices? What is the mathematical definition of P_C and P_B?

b) Write down the total differential of your savings, in terms of B, C, P_B and P_C.

c) Use the equality of mixed partial derivatives (Clairut's theorem) to find a relationship between P_B, P_C, B and C. Write this relationship mathematically, and also describe in words what it means.

d) Solve for the total differential of your net worth. Once again use Clairut's theorem considering second derivatives of W to find a different partial derivative relationship between P_B, P_C, B and C.

$$W \equiv S + P_B B + P_C C$$

Problem 2.3: Isowidth and isoforce stretchability In class on Tuesday, you measured the isowidth (or "Corinne") stretchability and the isoforce stretchability of your systems in the black boxes. We found that for some systems these were very different, while for others they were identical.

Show that the ratio of isowidth stretchability to isoforce stretchability is the same for both directions of a given system, i.e.:

$$\frac{\left(\frac{\partial x}{\partial F_x}\right)_y}{\left(\frac{\partial x}{\partial F_x}\right)_{F_y}} = \frac{\left(\frac{\partial y}{\partial F_y}\right)_x}{\left(\frac{\partial y}{\partial F_y}\right)_{F_x}} \tag{1}$$

[1] Yes, this is ridiculous. It would be slightly less ridiculous if we were talking about nations and commodities, but also far less humourous.

Hint You will need to make use of the *cyclic chain rule*, which we will derive on Thursday:

$$\left(\frac{\partial A}{\partial B}\right)_C = -\left(\frac{\partial A}{\partial C}\right)_B \left(\frac{\partial C}{\partial B}\right)_A \tag{2}$$

Hint You will also need the *ordinary chain rule*, which we will also derive on Thursday:

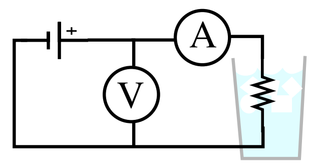
$$\left(\frac{\partial A}{\partial B}\right)_D = \left(\frac{\partial A}{\partial C}\right)_D \left(\frac{\partial C}{\partial B}\right)_D \tag{3}$$

Homework 3
Ice Calorimetry Lab

In this lab, we will be measuring how much energy it takes to melt ice and heat water.

Materials:

- Styrofoam cup
- Heating element
- Scale
- 2 digital multimeters
- Temperature guage
- Ice and water



The setup You will put some mass of ice (about 50g) and ice-cold water (about 150g) into your styrofoam cup. Use the scale to record the mass of the ice and water as you add them to the cup. Finally, add your ice-cold heating element and thermometer through the lid of the cup.

Collect data We will be measuring the temperature of the water and the power dissipated in the heating element (which is just a resistor). Thus we can find out how much energy was added to the water, and how this changes the temperature. In order to keep the temperature measurement reasonable, we will need to periodically stir the cup and heat it moderately slowly.

You will be collecting temperature data using the computer, so before you turn on the heater, you should make sure the computer is taking data. Turn on the heater, and write down the time you do so as well as the current and voltage, from which you can find the power dissipated in the resistor. If the current or voltage changes during the course of the experiment, take note of the new values—and the time.

Problem 3.1: Plot your data I Plot the temperature versus total energy added to the system (which you can call Q). To do this, you will need to integrate the power. Discuss this curve and any interesting features you notice on it.

Problem 3.2: Plot your data II Plot the heat capacity versus temperature. This will be a bit trickier. You can find the heat capacity from the previous plot by looking at the slope.

$$C_p = \left(\frac{\partial Q}{\partial T}\right)_p \tag{1}$$

This is what is called the *heat capacity*, which is the amount of energy needed to change the temperature by a given amount. The p subscript means that your measurement was made at constant pressure. This heat capacity is actually the total heat capacity of everything you put in the calorimeter, which includes the resistor and thermometer.

Problem 3.3: Specific heat From your plot of $C_p(T)$, work out the heat capacity per unit mass of water. You may assume the effect of the resistor and thermometer are negligible. How does your answer compare with the prediction of the Dulong-Petit law?

Problem 3.4: Latent heat of fusion

a) What did the temperature do while the ice was melting? How much energy was required to melt the ice in your calorimeter? How much energy was required per unit mass? per molecule?

b) The change in *entropy* is easy to measure for a reversible isothermal process (such as the slow melting of ice), it is just

$$\Delta S = \frac{Q}{T} \tag{2}$$

where Q is the energy thermally added to the system and T is the temperature in Kelvin. What is was change in the entropy of the ice you melted? What was the change in entropy *per molecule*? What was the change in entropy per molecule divided by Boltzmann's constant?

Problem 3.5: Entropy for a temperature change Choose two temperatures that your water reached (after the ice melted), and find the change in the entropy of your water. This change is given by

$$\Delta S = \int \frac{đQ}{T} \tag{3}$$

$$= \int_{t_i}^{t_f} \frac{P(t)}{T(t)} dt \tag{4}$$

where $P(t)$ is the heater power as a function of time and $T(t)$ is the temperature, also as a function of time.

Homework 5

Problem 5.1 (practice): Power from the ocean It has been proposed to use the thermal gradient of the ocean to drive a heat engine. Supoose that at a certain location the water temperature is 22^oC at the ocean surface and 4^oC at the ocean floor.

a) What is the maximum possible efficiency of an engine operating between these two temperatures?

b) If the engine is to produce 1 GW of electrical power, what minimum volume of water must be processed every second? Note that the heat capacity of water $C_p = 4.2$ Jg^{-1}K^{-1} and the density of water is 1 g cm^{-3}, and both are roughly constant over this temperature range.

Problem 5.2: Power plant on a river At a power plant that produces 1 GW ($10^9 watts$) of electricity, the steam turbines take in steam at a temperature of 500^oC, and the waste energy is expelled into the environment at 20^oC.

a) What is the maximum possible efficiency of this plant?

b) Suppose you arrange the power plant to expel its waste energy into a chilly mountain river at 15^oC. Roughly how much money can you make in a year by installing your imporved hardware, if you sell the addtional electricity for 5 cents per kilowatt-hour?

c) At what rate will the plant expel waste energy into this river?

d) Assume the river's flow rate is $100m^3/s$. By how much will the temperature of the river increase?

e) To avoid this "thermal pollution" of the river the plant could instead be cooled by evaporation of river water. This is more expensive, but it is environmentally preferable. At what rate must the water evaporate? What fraction of the river must be evaporated?

Problem 5.3: Bungee A physics major carefully measures the tension in a Bungee cord over a range of temperatures from room temperature to the boiling point of water. She examines her data carefully and finds that the tension in the cord is very well approximated by

$$\tau = \left(a - be^{-T/T_1}\right)\tan\left(\frac{\pi L}{2L_M}\right)$$

where L is the length that the cord is stretched beyond its relaxed length, and a, b, T_1 and L_M are positive constants.

She then places the relaxed cord ($L = 0$) in a calorimeter and measures the heat capacity over the same range of temperatures and finds that

$$C_L = \alpha + \gamma e^{-T/T_1}$$

where α and γ are two additional positive constants, and T_1 is the same value found in the previous experiment.

a) Sketch the tension τ versus the stretch L, and the heat capacity C_L versus the temperature T.

b) Find the change in free energy

$$\Delta F = F(T, L) - F(T, \tfrac{1}{2}L_M)$$

where T_R is room temperature.

c) Solve for the change in entropy $S(T, L) - S(T, \frac{1}{2}L_M)$ at an arbitrary temperature and length.

d) Solve for the change in internal energy $U(T, L) - U(T, \frac{1}{2}L_M)$ at an arbitrary temperature and length.

Problem 5.4: Using the Gibbs free energy You are given the following Gibbs free energy:

$$G = -kTN \ln\left(\frac{aT^{5/2}}{p}\right)$$

where a is a constant (whose dimensions make the argument of the logarithm dimensionless).

a) Compute the entropy.

b) Work out the heat capacity at constant pressure C_p.

c) Find the connection among V, p, N, and T, which is called the equation of state.

d) Compute the internal energy U.

Problem 5.5: Free expansion The internal energy is of any ideal gas can be written as

$$U = U(T, N) \tag{1}$$

meaning that the internal energy depends only on the number of particles and the temperature, but not the volume.[1] The ideal gas law

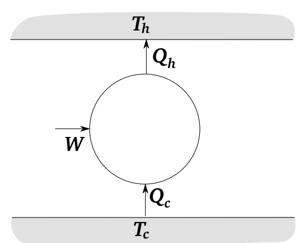
$$pV = Nk_BT \tag{2}$$

defines the relationship between p, V and T. You may take the number of molecules N to be constant. Consider the free adiabatic expansion of an ideal gas to twice its volume. "Free expansion" means that no work is done, but also that the process is also neither quasistatic nor reversible.

a) What is the change in temperature of the gas?

b) What is the change in entropy of the gas? How do you know this?

Problem 5.6 (challenge): Heat pump A heat pump is a refridgerator (or air conditioner) run backwards, so that it cools the outside air (or ground) and warms your house. We will call Q_h the amount of heat delivered to your home, and W the amount of electrical energy used by the pump.



a) Define a coefficient of performance γ for a heat pump, which (like the efficiency of a heat engine) is the ratio of "what you get out" to "what you put in."

b) Use the second law of thermodynamics to find an equation for the coefficient of performance of an ideal (reversible) heat pump, when the temperature *inside* the house is T_h and the temperature *outside* the house is T_c. What is the efficiency in the limit as $T_c \ll T_h$?

c) Discuss your result in the limit where the indoor and outdoor temperatures are close, i.e. $T_h - T_c \ll T_c$. Does it make sense?

d) What is the ideal coefficient of performance of a heat pump when the indoor temperature is 70°F and the outdoor temperature is 50°F? How does it change when the outdoor temperature drops to 30°F?

[1] This relationship happens to be linear at low temperatures, where "low" is defined relative to the energy of the excited states of the molecules or atoms.

Homework 7

Problem 7.1: Hot metal A 1 cm^3 cube of hot metal is thrown into the ocean; several hours pass.

a) During this time does the entropy of the metal increase, decrease, remain the same, or is this not determinable with the given information? Explain your reasoning.

b) Does the entropy of ocean increase, decrease, remain the same, or is this not determinable with the given information? Explain your reasoning.

c) Does the entropy of the metal plus the ocean increase, decrease, remain the same, or is this not determinable with the given information? Explain your reasoning.

Problem 7.2: Isothermal and adiabatic compressibility The isothermal compressibility is defined as

$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \tag{1}$$

K_T is be found by measuring the fractional change in volume when the the pressure is slightly changed with the temperature held constant. In contrast, the adiabatic compressibility is defined as

$$K_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S \tag{2}$$

and is measured by making a slight change in pressure without allowing for any heat transfer. This is the compressibility, for instance, that would directly affect the speed of sound. Show that

$$\frac{K_T}{K_S} = \frac{C_p}{C_V} \tag{3}$$

Where the heat capacities at constant pressure and volume are given by

$$C_p = T\left(\frac{\partial S}{\partial T}\right)_p \tag{4}$$

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V \tag{5}$$

Problem 7.3: Boltzmann ratio At low temperatures, diatomic molecule can be well described as a *rigid rotor*. The Hamiltonian of such a system is simply proportional to the square of the angular momentum

$$H = \frac{1}{2I}L^2 \tag{6}$$

and the energy eigenvalues are

$$E_{lm} = \hbar^2\frac{l(l+1)}{2I} \tag{7}$$

a) What is the energy of the ground state and the first and second excited states of the H_2 molecule?

b) At room temperature, what is the relative probability of finding a hydrogen molecule in the $l = 0$ state versus finding it in any one of the $l = 1$ states?
i.e. what is $P_{l=0,m=0}/\left(P_{l=1,m=-1} + P_{l=1,m=0} + P_{l=1,m=1}\right)$

c) At what temperature is the value of this ratio 1?

d) At room temperature, what is the probability of finding a hydrogen molecule in any one of the $l = 2$ states versus that of finding it in the ground state?
i.e. what is $P_{l=0,m=0}/\left(P_{l=2,m=-2} + P_{l=2,m=-1} + \cdots + P_{l=2,m=2}\right)$

Problem 7.4 (challenge): A plastic rod When stretched to a length L the tension force τ in a plastic rod at temperature T is given by its Equation of State

$$\tau = aT^2(L - L_o)$$

where a is a positive constant and L_o is the rod's unstretched length. For an unstretched rod (i.e. $L = L_o$) the specific heat at constant length is $C_L = bT$ where b is a constant. Knowing the internal energy at T_o, L_o (i.e. $U(T_o, L_o)$) find the internal energy $U(T_f, L_f)$ at some other temperature T_f and length L_f.

a) Write $U = U(T, L)$ and take the total derivative dU.

b) Show that the partial derivative $(\partial U/\partial L)_T = -aT^2(L - L_o)$.

c) To integrate the resulting differential equation Line Integrate dU *very carefully* in the T, L plane, keeping in mind that $C_L = bT$ holds *only* at $L = L_o$.

Appendix C: Partial Derivative Machine, 2013 Version

Description

The Partial Derivative Machine is an apparatus consisting of a central spring system that can be stretched via four strings extending outward from the center (See Fig. 5). Alternative central systems can be used for this activity including a loop of string, a piece of spandex, and other combinations of springs (see Fig. 6). This central system is on a large piece of particle board which features a pulley on two adjacent corners (Corners C and D in Fig. 5), and a knob on all four corners (See Fig. 7). By tightening the knobs at A and B (See Fig. 5), one can hold the system in place while adding weights to the hanging strings, allowing one to manipulate the state of the system.

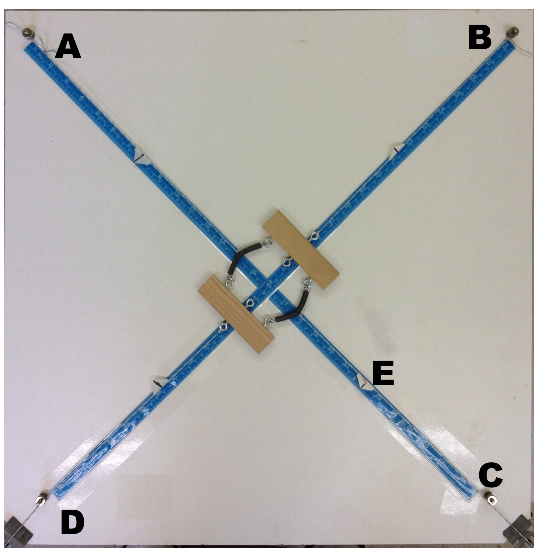


FIG. 5. The PDM, (A/B)Corners with only knob, (C/D)Corners with pulley, (E)Measuring Flag.

In order to more easily measure the stretching of the system, a measuring tape is placed on the board parallel to each string and flags are added to the strings (Example labeled E in Fig. 5). By labeling the axis from corner B to corner D as the "X-axis" and the axis from

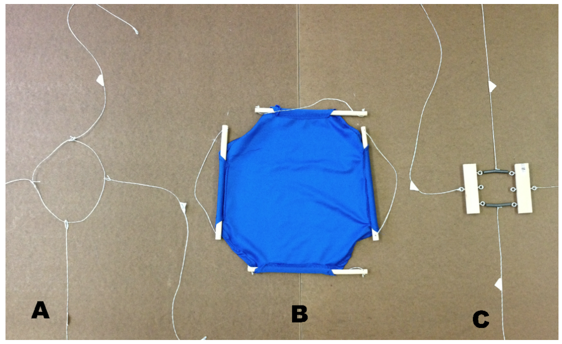


FIG. 6. Central Systems: (A)Loop of String, (B)Piece of Spandex, and (C)Spring System.

corner A to corner C as the "Y-axis" the instructor is able to define four quantities for this experiment:

1. x, the distance between the flags on the X strings

2. y, the distance between the flags on the Y strings

3. F_x, the tension in the X oriented strings

4. F_y, the tension in the Y oriented strings

There are two conditions under which the system can be manipulated. The first method involves tightening the knob on corner C or D to pin a third string, thereby fixing x or y, and then increasing the mass on the freely hanging string. For example, pinning the knob at C would fix y, then adding weight to the X string would increase x, F_x, and F_y.

Alternatively, one can leave both the X and Y strings free and add weights to one or both. In doing so, placing weight on a string causes the system to stretch in one direction while compressing in the other direction as the system balances the forces. For example, adding weight to the X string would cause x and F_x to increase while F_y stays constant and y decreases.

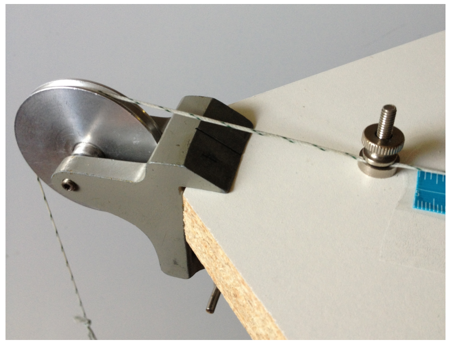


FIG. 7. Corner with Pulley and Knob (C/D in Fig. 5).

It is important to note that as weights are added it is not uncommon for the system to shift from its centered position. In order to keep F_x and F_y orthogonal as the system is shifted, students are told to temporarily loosen knobs at A and B to recenter the system between measurements. If done correctly, this action is only a translation of the system. Thus it does not result in stretching or compressing the system and does not impact any of the measurements students are instructed to make.

Use of the PDM in the Classroom, 2013

Introduction

When first introduced to the Partial Derivative Machines, the central system was hidden from students through the use of a "black box" (See Fig. 8). With only the knowledge that there were four strings extending from this box, students were asked to determine:

- The properties of the system that can be controlled.
- The properties of the system that can be measured.
- The number of independent properties of the system.

Students worked briefly in groups of 3 students to answer this prompt and then were brought back together for a class discussion. During the wrap-up discussion, students listed a number of controllable properties including the position of the central system relative to the center of the board, the forces applied to the system, and the amount the system was stretched in either direction. Students decided it was possible to measure x and y by taking values for the positions of the flags, and to measure F_x and F_y by noting the mass hung from the relevant string.

FIG. 8. Old PDM with “Black Box” included.

Many students did not realize however that the tension in a particular string is not equivalent to the weight hung from that string if the corresponding knob is locked since the mass becomes irrelevant when the string is pinned down. Most students also determined that only two of these properties could be controlled independently and that manipulating a pair of parameters caused a responsive change in the other parameters.

Finding a simple derivative

Once students were familiar with the machine, they were asked in a second exercise to find $\frac{\partial x}{\partial F_x}$ and had to consider that there were two possible options: $\left(\frac{\partial x}{\partial F_x}\right)_y$ and $\left(\frac{\partial x}{\partial F_x}\right)_{F_y}$. As an introduction, the instructor defined the concept of stretchability as it relates to the system and distinguished between the "isowidth" (constant y) and "isoforce" (constant F_y) stretchabilities.

After collecting data sets, plotting results, and calculating numerical values for both quantities students were asked to present their results to the class. The focus of the presentation was not to provide the class with numerical values, but to explain the techniques used to both measure and calculate the necessary information. The approach of some groups was to take a few measurements of the form (F_x, x) and approximate the derivative with the quantity $\frac{\Delta x}{\Delta F_x}$. Other groups chose instead to plot x as a function of F_x.

Due to the different systems under the black boxes, the numerical values for the "isowidth" stretchability and "isoforce" stretchability varied widely from group to group. The relationship between x and F_x also varied from system to system — some groups found a linear relationship while others found that the plot was clearly nonlinear.

After these presentations and discussion of the results, students removed the "black box" to see the central systems. Students then walked around the classroom observing other groups' systems to see how each apparatus was different. This allowed for discussion of why particular systems behaved as they did and why particular variables were dependent or independent of each other for each system.

These observations were followed by a whole-class discussion. The instructor asked students to consider if this activity was consistent with or contradicted the idea that one takes a partial derivative while holding "everything else" constant. Next, the class revisited the number of independent variables and which could be set simultaneously.

It was not obvious to some students that y and F_y were relevant quantities when changing x and F_x. To address this concern the instructor conducted a demonstration making use of the piece of spandex (B in Fig. 6). Having one student grab a pair of opposite handles and hold them a fixed distance apart, a second student was instructed to stretch the spandex in the other direction, which simulated measuring $\left(\frac{\partial x}{\partial F_x}\right)_y$. It then became abundantly clear to the first student that in order to maintain a constant y it was necessary for F_y to increase as F_x increased. We have repeatedly found that the kinesthetic effect of feeling the force increase in this demonstration helps people notice that the force and displacement in the two directions are coupled.

Integrated Lab

As a preface to a major activity associated with the Partial Derivative Machines, students were given a review lecture on:

- Calculating changes in potential energy, ΔU, as the work, W, done on the system
- Finding potential energy of stretched springs
- Work as the integral of force, $W = \int F dx$

After this review they proceeded to conduct a laboratory experiment. The primary task was to measure the potential energy stored in the spring system of the Partial Derivative Machine, however a process to determine this function was not explicitly given. The review of work, potential energy, and springs prior to data collection was designed to help students make the connection that the potential energy could be obtained from the work done on the system. Since the system was now two dimensional, using $W = \int \vec{F} \cdot d\vec{r}$ required finding the work done on the system in both the X and Y directions.

One possible solution method that determines all necessary information is:

1. Starting at a particular $x = x_o$, where $\Delta x = 0$, take measurements of y while changing F_y in uniform steps, *e.g.,* $0.05kg \times 9.81m/s^2$.

2. Set subsequent x values by loosening knob D, incrementing F_x by small uniform steps, and then tightening knob D.

3. Repeat step 1 for each new fixed x value.

4. Using the data and numerical integration of $F_x\,dx$ and $F_y\,dy$, approximate the value of $U(x, y)$.

This process gave students the data needed to get from any state (x_1, y_1) to a different state (x_2, y_2), provided each corresponded to a state generated during the steps outlined above. To verify path independence one would need to conduct a similar process, now measuring x for fixed y values while varying F_x (changing F_x and F_y by the same increments used above). This lab also provided students practice distinguishing between fixed y and fixed F_y processes and the relevance of each to particular measurements.

Connections to Thermodynamics

The integrated lab was designed to allow students to reach the conclusion that there are two ways to manipulate the potential energy of the system, each corresponding to a particular force (F_x or F_y) and distance (x or y). The students were able to see the pairing of these quantities through the differential expression for work,

$$dU = F_x dx + F_y dy$$

which is a mechanical equivalent of the thermodynamic identity, using conjugate pairs of forces and distances rather than the canonical thermodynamic conjugate pairs: temperature and entropy, pressure and volume.

The instructor then introduced the concept of the potential energy of the system, U, as a state function determined by x and y, a function whose total differential is given by:

$$dU = \left(\frac{\partial U}{\partial x}\right)_y dx + \left(\frac{\partial U}{\partial y}\right)_x dy$$

From Eqs. C and C the instructor extracted definitions for the forces as partial derivatives of the potential energy, a result which should be familiar from both classical mechanics and E & M.

Using these expressions for F_x and F_y students were asked to express $\left(\frac{\partial F_x}{\partial y}\right)_x$ and $\left(\frac{\partial F_y}{\partial x}\right)_y$ as derivatives of U. From these new derivative expressions, and Clairaut's theorem (the order of mixed partials does not matter), students found a Maxwell Relation for their system that they could experimentally verify.

During the remaining three contact hours of the Interlude the instructor led students through additional mathematical techniques relevant to thermodynamics including partial derivative manipulations, the cyclic chain rule, and Legendre Transforms, using the physical parameters of the PDM. These exercises allowed students to practice these math methods and understand their results in the context of tangible experimental measurements.