Rubber Band Pre-Lab

In this lab, we will measure the tension of a rubber band stretched to a fixed length, as a function of temperature. By measuring this at several similar lengths, we will be able to determine the change in internal energy U and entropy S for a rubber band that is stretched at fixed temperature.

Materials:

- One rubber band
- Tube
- Stopper with hook
- Vernier force guage
- Vernier temperature guage
- Several clamps
- Boiling water
- Ice
- Pan

Background

The inexact differential of work for a system such as a rubber band that is stretched in just one direction is

$$đW = -\tau dL.$$

This follows naturally from the definition of work as force dotted with distance—provided one takes into account the sign convention for tension, which is opposite that of pressure. Thus the thermodynamic identity is:

$$dU = TdS + \tau dL$$

We could use the thermodynamic identity directly, but since we are working at constant T, it is more helpful to consider the Helmholtz free energy

$$F \equiv U - TS$$

The total differential of F is

$$dF = \tau dL - SdT$$

from which we can extract a Maxwell relation:

$$\frac{\partial^2 F}{\partial L \partial T} = \frac{\partial^2 F}{\partial T \partial L}$$

$$\left(\frac{\partial}{\partial L}\left(\frac{\partial F}{\partial T}\right)_L\right)_T = \left(\frac{\partial}{\partial T}\left(\frac{\partial F}{\partial L}\right)_T\right)_L$$

$$-\left(\frac{\partial S}{\partial L}\right)_T = \left(\frac{\partial \tau}{\partial T}\right)_L$$

By measuring the variation of *tension* with *temperature* at fixed *length*, we can find out how *entropy* will change when the *length* is changed at fixed *temperature*! At the same time, a measurement of the tension will tell us how the free energy will vary under the same isothermal stretch:

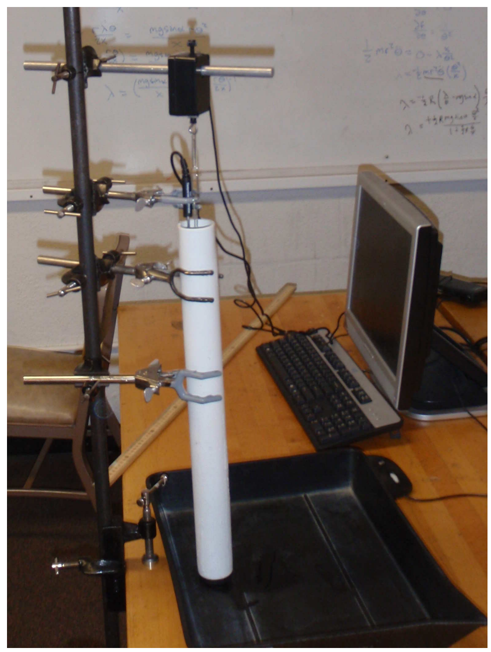
$$\tau = \left(\frac{\partial F}{\partial L}\right)_T$$

Thus, provided we know τ and $\partial\tau/\partial T_L$ for our rubber band, as a function of the length, we can integrate to find ΔS, ΔF and ΔU for an isothermal stretch.

The setup

You will stretch your rubber band between a force meter and a hook in the stopper in the bottom of a pipe. If you attach the rubber band to the force meter by means of a chain of paper clips, then you can ensure that when the pipe is filled with water, the rubber band is completely immersed.

You will need to insert a thermometer probe into the top of the pipe in order to measure the temperature of your water—and thus the temperature of your rubber band.



Collect data

During the experiment you will pour water of various different temperatures into the pipe in order to heat up your rubber band, and record the tension as a function of length. When changing to a different temperature, you will need to empty your pipe and refill it. Each time, you should measure the temperature at the top and bottom to ensure that the water is well mixed.

Conclusions and Questions

Please answer the following questions. As always, show your work. In each case, discuss whether your result agrees with your predictions. If it disagrees, please attempt to explain what was wrong with your reasoning.

Tension vs. temperature Plot the tension versus temperature for each of your lengths on the same plot.

Tension vs. length Plot the tension versus length for a few temperatures.

$\left(\frac{\partial S}{\partial L}\right)_T$ vs. length Plot $\left(\frac{\partial S}{\partial L}\right)_T$ versus length for the same set of temperatures you chose for Question .

Isothermal stretch Pick a temperature and a range of lengths for which you have clean data.

1. What is the change in free energy for an isothermal stretch at this temperature from the smallest length to the largest?
2. What is the change in entropy?
3. What is the change in the internal energy?
4. What is Q?
5. What is W?
6. What is $|Q/W|$?

Adiabatic stretch What additional experiments would you need to perform in order to answer Question for a stretch that is adiabatic rather than isothermal?

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