Two vs. Three Dimensions

The traditional approach to multivariable calculus emphasizes functions of two variables as a way to learn about functions of several variables. There is even an excellent text [2] which works exclusively in two dimensions. Yet the real world is made up of functions of three variables.

1 Graphs

Many students view a graph as the fundamental representation of a function. But you can only graph functions of one or two variables! While graphing techniques have their place, we believe it is essential to place at least as much emphasis on techniques which do generalize to functions of three variables, such as the use of contour diagrams and color.

When we begin discussing functions of more than one variable, we do a classroom demonstration which involves projecting a spreadsheet showing numerical data directly on a whiteboard. A good example to use is the temperature on a 2-dimensional flat surface. We ask students to come to the board and draw level curves directly through the numerical data. We then replace the numerical table by a colored contour plot constructed by the spreadsheet from the data. [1] In many spreadsheet programs, this contour plot is really a 3-dimensional graph, seen head on, which can then be rotated live to produce the graph. This demonstration provides a visual connection between numerical data, contour diagrams, the use of color, and the graph, which furthermore emphasizes the fact that the graph requires an extra dimension. We follow this with a (usually lively) class discussion

[1] We often follow this demonstration with one involving the gradient, superimposing a plot of the gradient on the contour diagram.

about representing a function of three variables, such as the temperature in the room.

2 Potential Functions

Many calculus texts treat conservative vector fields in two and three dimensions as separate problems. In two dimensions, one checks the mixed partial derivatives of the components; in three dimensions, one checks the curl. Some authors attempt to connect these two cases by introducing the "scalar curl" in the 2-dimensional case; we feel this only makes things worse.

There are two separate ideas here, potential functions and curl, and we choose to separate them. When finding potential functions, there is no need to check derivatives at all, and in particular no need to compute the curl. Our method for finding potential functions, which is described in more detail in [3], involves integration, not differentiation. This method works equally well in two and three dimensions; we make no effort to divide these into separate cases.

3 Green's Theorem

Why teach Green's Theorem? The best answer I have heard to this question is that one might as well teach Green's Theorem since there's not going to be time to cover Stokes' Theorem! Yes, Green's Theorem is important in its own right, it has for instance applications to the theory of complex variables. But Green's Theorem *is* Stokes' Theorem; it takes very little effort to get from one to the other.

We therefore skip Green's Theorem entirely, mentioning it only incidentally as the first, and most important, step when proving Stokes' Theorem.

Bibliography

[1] Tevian Dray and Corinne A. Manogue, *Spherical Coordinates*, College Math. J. **34**, 168–169 (2003).

[2] Robert Osserman, **Two-Dimensional Calculus**, Harcourt, Brace, and World, New York, 1968.

[3] Tevian Dray & Corinne A. Manogue, *The Murder Mystery Method for Determining Whether a Vector Field is Conservative*, College Math. J. **34**, 238–241 (2003).