Cognitive Development At The Middle-Division Level

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Abstract. One of the primary goals, as students transition from the lower-division to upper-division courses is to facilitate the cognitive development needed for work as a physicist. The Paradigms in Physics curriculum (junior-level courses developed at Oregon State University) addresses this goal by coaching students to coordinate different modes of reasoning, highlighting common techniques and concepts across physics topics, and setting course expectations to be more aligned with the professional culture of physicists. This poster will highlight some of the specific ways in which we address these cognitive changes in the context of classical mechanics and E&M.

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INTRODUCTION

Eleven years ago, the Paradigms in Physics Project at Oregon State University reformed the entire upper-division curriculum for physics and engineering physics majors. This involved both a rearrangement of content to better reflect the way professional physicists think about the field and also the use of a number of interactive pedagogies that place responsibility for learning more firmly in the hands of the students. The junior year now consists of short case-studies of paradigmatic physical situations which span two or more traditional subdisciplines of physics. One of the main goals of these courses is for students to become more sophisticated problem-solvers. The classroom activities and content-ordering and are structured to support this goal.[1]

In this article, we give a detailed description of one Paradigms activity in order to illustrate some of the ways that these activities are designed to scaffold the development of students' cognitive capacities. We present a task analysis of the problem focusing on modes of cognition students must employ, discuss student difficulties and suggest some ways in which our curriculum development work might interface with physics education research.

ACTIVITY DESCRIPTION

In the activity "Electrostatic Potential of Two Point Charges", students work in small groups (3 or 4 students) to solve the following problem:

> "Two charges $+Q$ and $+Q$ are placed on a line at $x=+D$ and $x=-D$, respectively. Find the electrostatic potential everywhere in space. What is the fourth order approximation of the electrostatic potential, V, valid on the x-axis, for $|x|>>D$".

Each group completes a slightly different version of this problem (different groups consider different regions of space for doing the approximation and some groups consider the potential of two opposite charges).

TASK ANALYSIS

In order to solve the problem correctly, students may do the following (although students may combine steps and often do not proceed in this order):

- (CF) Start with an "iconic" equation—the potential due to a point charge.
$$V=\frac{kq}{r}.$$
- (TR) Recognize that the r in the iconic equation represents the distance between the source and observation points, $|\vec{r}-\vec{r}_i|$.
- (CF) Recognize that the superposition principle applies.
$$V(\vec{r})=\frac{1}{4\pi\varepsilon_0}\sum_i\frac{q_i}{|\vec{r}-\vec{r}_i|}$$
- (AP) Choose a coordinate system and draw a diagram.

- (TR) Choose a coordinate label $(x,0,0)$ for the point at which the potential will be evaluated.
- (TR, AM) Evaluate the distances in the denominator for the specific case.

$$V(x)=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{\sqrt{(x+D)^2}}+\frac{Q}{\sqrt{(x-D)^2}}\right)$$

- (AM) Recognize from the geometry that the denominators should be expressed as absolute values, especially when x is negative.

$$V(x)=\left(\frac{Q}{|x+D|}+\frac{Q}{|x-D|}\right)$$

- (RP) Recognize that the denominators have something to do with a known series.
- (AM) Decide what to factor out to put the denominators in the form of “one plus something small and dimensionless.”

$$V(x)=\frac{1}{4\pi\varepsilon_0}\frac{Q}{|x|}\left(\left(1+\frac{D}{x}\right)^{-1}+\left(1-\frac{D}{x}\right)^{-1}\right)$$

- (AM) Implement known mathematics from a memorized power series. Simplify, group terms.

$$V(x)=\frac{1}{4\pi\varepsilon_0}\frac{2Q}{|x|}\left(1+\left(\frac{D}{x}\right)^2+\dots\right)$$

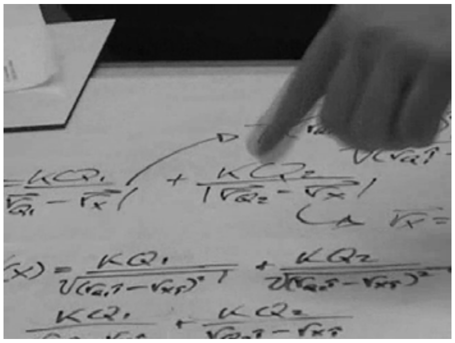

FIGURE 1. Students work on the Discrete Charges activity in groups of three on medium white boards.

MODES OF COGNITON

It is informative to do a task analysis employing a framework that identifies different modes of cognition. Redish and Hammer [2] have introduced the following nine cognitive modes that they teach students explicitly to recognize :

- (CF) Choosing foothold ideas, choosing an iconic formula
- (RS) Restricting the scope
- (SM) Sensemaking
- (TR)Translating representations
- (SC) Seeking Coherence
- (SI) Shopping for ideas
- (PR) Probing and refining intuitions
- (IG) Playing the implications game
- (EN) Employing a safety net.

To describe problem-solving activities, we’ve included four additional modes in our task analysis:

- (FF) Fleshing out formulas – adding mathematical detail to an iconic formula.
- (AM) Applying learned mathematics – executing algebra, calculus, etc.
- (RP) Recognizing patterns – identifying a place where a known result or a familiar set of steps can be employed in a solution.
- (AP) Applying a principle to a specific case – exploiting geometry or symmetry to simplify a general formula.

PEDAGOGICAL CONTENT KNOWLEDGE

Pedagogical Content Knowledge (PCK) is the kind of knowledge a teacher has about how students interact with a particular topic. Listed here are very common student difficulties that come up as students are working on the activity, together with our assessment of the cognitive capacities that are required to successfully address each problem.

Students often claim they “can’t get started.” Many times, the difficulty lies somewhere in the early process of using geometry to translate an abstract, coordinate-independent, algebraic representation to a coordinate-dependent, algebraic representation on which the students can “do math.” This is a complex process that may require any of a number of different cognitive capacities (CF, RS, SM, TR, SI, FF, RP AP).

Most students either leave out or drop the absolute value signs in the algebraic expression for the distances between the sources and the observation point, ignoring the case where x is negative. This topic can be a source of rich class discussion during the whole-class wrap-up, may be prompted by an apparent inconsistency between the symmetry of the problem and the evenness or oddness of the powers that appear in the expansion (RP, AM).

Students have the most difficulty with the series expansion. Problems range from believing that the only way to find a series is through the successive differentiations in Taylor’s formula, the failure to recognize that $\frac{1}{|x+D|}=|x+D|^{-1}$, not recognizing that $(1+x)^p$ is a common power series, and not knowing how to put $\frac{1}{|x+D|}$ in the form $\alpha(1+\varepsilon)^p$ (RP, AM).

WRAP-UP

In a whole-class wrap-up discussion, students compare examples from different groups (each group finds a series expansion in a different region of space), compare their results with known limiting examples, discuss the physical implications of higher-order terms in the series expansion, explore the role of symmetry in the problem and solutions. The wrap-ups are also a wonderful opportunity for students to practice presenting their ideas clearly, learn how to speak loudly, learn conventional terminology for their concepts, etc. This discussion may take as long as, or longer than, the group problem solving. We note especially that the number of cognitive modes employed by students in this part of the activity is much greater than in any other part: (RP, SM, SN, AP, TR, SC, PR, IG, EN). Typically, class sizes in the upper-division, compared to the lower-division, are more likely to allow for faculty to hold these wrap-ups. We encourage faculty to take advantage of the rich opportunity to scaffold cognitive development.

PREPARATORY ACTIVITIES

None of our activities stand in isolation; instead they occur in sequences that allow students to explore different aspects of each paradigmatic physical situation. These different aspects require different pedagogical strategies on the part of the instructor and reinforce the development of different cognitive capacities on the part of the student. Activities that lead up to the one described above include:

- Potential of a point charge: Students recall the formula for the potential due to a point charge, recording their attempts on individual white boards. Class discussion focuses on strategies to choose the correct formula.

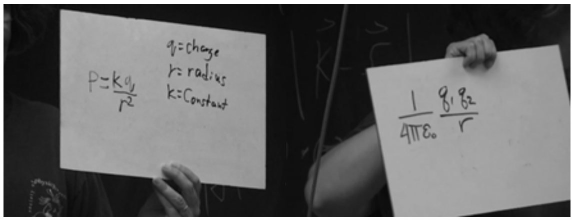

FIGURE 2. Potential of a point charge.

- Star Trek: Using a Star Trek scenario as a premise, students discuss how to specify the distance between two objects (Captain Kirk and Mr. Spock) recognizing that the positions of each must be specified with reference to an origin.

FIGURE 3. Star Trek.

- Power Series: Students use a computer algebra package to plot the first several terms of a power series expansion and visually compare their approximation with a plot of the function.

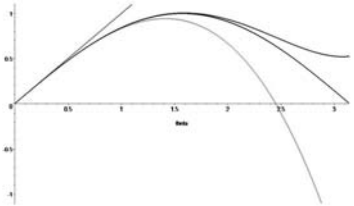

FIGURE 4. Power Series.

FOLLOW-UP ACTIVITIES

Activities that follow-up on the concepts originally developed in the one described above include:

- Drawing equipotential surfaces: Students are asked to draw equipotential surface for various charge distribution on their whiteboards.

FIGURE 5. Drawing equipotential surfaces.

- Visualizing potentials: Using a computer algebra system, students explore different ways of visualizing a scalar field in three dimensions.

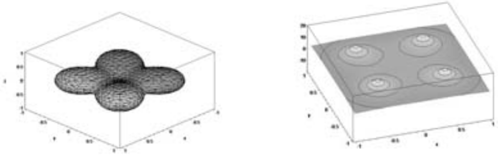

FIGURE 6. Visualizing potentials.

- Acting out charge densities: Students place themselves around the classroom to model various charge density distributions (linear, surface, and volume) while building their conceptual understanding of the idealizations involved in generalizing from discrete to continuous charge distributions.

FIGURE 7. Acting out charge densities.

- Potential of a ring of charge: Students working is groups of three calculate the electrostatic potential due to a ring of charge. Compared to the Discrete Charges activity, they must now deal with integrals instead of sums. On a later day, they calculate the electric field due to a ring of charge and must then cope with a vector-valued integrand.

RESEARCH LENSES

Physics education research takes place in the context of a theoretical framework. Broadly speaking, the theoretical framework may use one of the following research lenses or ways of thinking about learning [3]:

- Behaviorism—describes learning by focusing on the behaviors of students. Anything that a student does is described in terms of behaviors, including thinking and learning. Behaviorists do not consider abstract constructs (such as the mind) in their analyses.
- Cognitivism—describes thinking by positing the existence of mental states that are manipulated during thinking. Cognitivists infer the structure of cognitive entities from experiments and observations of students.
- Situativism—describes knowing by considering that the actions of students are affected by the context (social, cultural, physical) in which the students' perceive themselves to be. Situativists view knowing as determined by both the person and the context. Learning is identified by students' increasingly effective performance across situations, rather than by an accumulation of knowledge.

Each of these lenses suggests different types of research questions: What mistakes to students typically make while formulating a solution? How do students coordinate algebraic statements with the geometry of the problem situation? How do students use the concepts of electrostatic potential, superposition, and power series expansions in their solutions? How can the activity be structured to support the development of these concepts? How do students use different representations in formulating their solutions? How does the compare-and-contrast structure of the activity affect student learning? How is students' discourse affected by the compare-and-contrast structure of the activity? How do students learn the professional norms of how to communicate about physics?

Choosing a research lens allows one to focus only on aspects of the activity of interest to the researcher, simplifying the analysis to a tractable project that can lead to a better understanding of how people learn physics. However, it is desirable for students to have a multifaceted experience that will offer challenges to all students. None of these lenses alone can adequately describe the richness of the classroom experience. The challenge for curriculum developers is to incorporate the results of physics education research from multiple lenses in order to provide rich experiences for students.

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REFERENCES

[1] More information about our activities and about the Paradigms program can be found on our wiki: *http://physics.oregonstate.edu/portfolioswiki*

[2] E. F. Redish and D. Hammer, *Am. J. Phys.* **77**, 629-642 (2009).

[3] J. G. Greeno, A. M. Collins, and L. B. Resnick, "Cognition and Learning," in *Handbook of Educational Psychology,* edited by D. Berliner and R. Calfee, New York: MacMillan, 1996, pp. 15-41.