Breaking a Big Problem into Manageable Pieces without Losing Sight of the Big Picture

The Students' Mindframe

In the best-case scenario, juniors in undergraduate physics will begin their junior year capable of successfully understanding and solving multi-step physics problems. Ideally, these students would have no significant holes in either their math or physics background and would already have the types of conceptual understanding that is measured by things such as concept inventories. Even given this seemingly rosy situation, only a very few of these juniors would have developed the depth of understanding needed to solve a problem such as finding the magnetic field in all space for a ring of current, without resorting to "pattern matching" by comparing a specific problem with a similar problem in the text.

There exists the rare extraordinarily capable student who has an exceptionally strong background and is absolutely determined to have a thorough understanding, and who has a life and course load that can accommodate spending many hours wading through cryptic references to concepts that are obvious to textbook authors, that could potentially succeed with this. However, the remaining majority will not have sufficient resources to meaningfully understand and succeed at solving complex problems without sufficient support.

Most students will have some holes in their math background. Most students will have resorted to pattern matching to solve at least some of the problems in their introductory physics courses. And most students will have no idea how to successfully approach such a difficult problem.

Students will use the resources they have and attempt to draw on past experience to solve new problems. Even students who genuinely want to understand and have the epistemological stance that they should be genuinely understanding the physics they are doing, will resort to other strategies when faced with an overwhelming situation.

The approach of many textbooks is to give students example problems that are sufficiently similar that students can look at these and learn how to do the new problem. The concern is that the "learning" from the example problem most frequently resembles weak understanding and pattern matching without any appreciation for subtleties or for why certain choices were made by the author for approaching the problem.

Students will often seek to find somewhere in which the problem is sufficiently structured that they can start cranking through the plugging and chugging that many students have become very good at. However, it is the setting up of the problem, and dealing with pieces of problems that are new and different, that physics majors will need to be successful. If the goal is to have students who are capable of solving complex problems, students need to learn to break a problem into manageable pieces and work through these pieces successfully without losing sight of the overall problem they are trying to solve.

Breaking the Problem into Pieces

For an experienced physicist looking at upper-division E and M problems, it often takes less than a minute to visualize the problem, consider the overall geometry and symmetries involved, and create a road map for approaching the problem, including envisioning many the pieces that will be needed to solve the problem. While, there is no magic way to have students instantly develop these abilities, there are ways to have students make meaningful advances toward this type of thinking.

Although there is more to problem solving than drawing a good picture and repeatedly referring to it, we have found that it is important to have students continually aware of how the geometry of a problem is related to its solution. Essentially, a good picture and a good understanding of the geometry, is a "necessary but not sufficient" condition for success.

In an anecdote from our own junior level E and M course, students were working in groups of three to solve for the electric potential in all space due to a ring of charge. Three of the groups made outstanding progress with only a minimal level of assistance from the instructor, while three groups made slow and halting progress, even with extensive assistance from the instructor. We have the fortune to have cameras mounted above each of the tables where the groups were working and were able to go back through the six recordings of the individual groups. What was most striking in just quickly scanning the videos was that the successful groups had all drawn a picture of the problem within the first two minutes and repeatedly pointed to the picture as they were working through parts of the problem. The three groups that had very limited success started by writing down lots of formulas or equations and trying to make progress using algebra and integrals before even drawing a picture.

Unfortunately, simply telling students to start by drawing a picture is not good enough for many students. Many students will draw a picture and then proceed to ignore it as they try to crunch through the problem by manipulating a sea of symbols in some prescribed fashion.

Referring back to a picture or a geometric understanding of the problem is critical in several stages including:

- understanding the question and the nature of the problem being asked
- choosing a coordinate system
- determining which laws or formulae are relevant
- setting up integrals
- evaluating integrals
- considering appropriate limiting cases
- checking to see if the final answer makes sense

Students will probably have little, if any, experience with doing any but the first and last steps. For example, students often need to be explicitly told that they can use their understanding of the geometry of the problem to help with evaluating the integral. When a professional physicist sees an integrand with a sea of symbols, they quickly try to assess which of these symbols represent quantities that remain constant during integration. Students often know that numbers like "G" are constants, but will fail to recognize that certain quantities that are variable in one context are constant in another. Students usually need guidance to use understanding of the geometry of the problem to recognize which things are variables and which things are constant.

Recognizing that utilizing symmetries can result in "variables" becoming constants for specific cases is important. It helps students to choose appropriate limiting cases, such as considering what happens to a field along a particular axis.

The Ring of Charge - One Approach to Building Understanding

In our case, we developed a series of five activities, four of which involved a ring of charge or ring of current. This creates a situation in which students are not simultaneously bombarded with new new physics concepts while simultaneously needing to wrestle with new geometries. With this sequence of activities, students deal with successively more complex problems within the context of a familiar geometry, and develop the understandings needed to successfully solve a problem like finding the magnetic field in all space due to spinning ring.

The first activity had students find the electric potential due to two point charges. After that a ring of charge was used to have students find the electric potential and then the electric field. Finally students were required to find the magnetic vector potential and the magnetic field for a spinning ring of charge. The details of these activities can be found in the E and M section of this wiki.

An additional benefit of using the same geometry for four consecutive problems is that students develop insights and understanding about the similarities and differences between electrostatic potential, electric field, magnetic vector potential and magnetic field. These differences can be lost when a new geometry is used for each new type of problem.

By the end of this series of five activities, students will have become proficient at using increasingly complicated power series and Laurent series expansions. In doing so they will have had to wrestle with "what is small" and what an expansion tells them about a physical situation. They will also have become comfortable with elliptic integrals and using both power series and Maple visualizations to help understand the results. In addition they will have repeatedly used geometric arguments and gone back and forth among physical thinking, geometric reasoning, and algebraic symbols.

Students come from a sequence of lower-division physics classes in which figuring out which formula to plug which numbers into can be a successful strategy for receiving a good grade in the course. Students frequently learn that they can be successful even if they ignore derivations and only focus on the resulting formulas for a variety of cases. To develop new habits of the mind, the old strategies need to be rendered ineffective in a context in which students are given sufficient scaffolding for them to be successful using new and unfamiliar ways of thinking.

This strategy fits the model of cognitive apprenticeship where the expert models thinking, students are coached and supported as they work through a task, and students have to articulate their knowledge, reasoning and problem-solving process. Each of these components is an important part of each of these activities.

Collectively these activities will be starting students down the road to thinking like a physicist. Students learn to unpack progressively more complicated problems into solvable pieces using geometric reasoning and mathematical tools. This moves students away from a plugging-into-formulas approach and starts building problem-solving strategies that will be far more useful to a future physicist.