Representations For A Spins-First Approach To Quantum Mechanics

Corinne Manogue*, Elizabeth Gire†, David McIntyre*, Janet Tate*

**Department of Physics, Weniger Hall 301, Oregon State University, Corvallis, OR 97331*
†Department of Physics, Manning Hall 421, University of Memphis, Memphis, TN 38152

Abstract. In the Paradigms in Physics Curriculum at Oregon State University, we take a spins-first approach to quantum mechanics using a java simulation of successive Stern-Gerlach experiments to explore the postulates. The experimental schematic is a diagrammatic representation that we use throughout our discussion of quantum measurements. With a spins-first approach, it is natural to start with Dirac bra-ket language for states, observables, and projection operators. We also use explicit matrix representations of operators and ask students to translate between the Dirac and matrix languages. The projection of the state onto a basis is represented with a histogram. When we subsequently introduce wave functions, the wave function attains a natural interpretation as the continuous limit of these discrete histograms or a projection of a Dirac ket onto position or momentum eigenstates. We are able to test the students' facility with moving between these representations in later modules.

Keywords: Quantum Mechanics, Multiple Representations, Upper-Division, Stern-Gerlach, Dirac Bra-Ket Notation
PACS: 01.40.Fk, 03.65.Xp

INTRODUCTION

Quantum mechanics is often considered to be abstract and this abstractness is deemed to cause confusion for students [1,2]. In the Paradigms in Physics Project at Oregon State University, we attempt to ameliorate this situation by explicitly introducing representations that focus student attention on the physical aspects of the things being represented. We also introduce a number of activities, including homework activities, which ask students to translate between representations [3].

In this article, we present many of the representations that we use in our courses in quantum mechanics and the reasons why we have chosen them. Where available, we also indicate the preliminary evidence for the effectiveness of these representations and discuss the occasional problems that we have noticed that arise from some of the representations.

The courses in the Paradigms Program are quite modular. An important aspect of this modularity is that faculty must buy in to the idea that it benefits weaker students if we know what representations have been introduced and agree to revisit representations over time. When many students use a representation introduced in one course spontaneously and appropriately in another setting in a later course, we take this as strong evidence that the representation is useful and that the concept represented has transferred.

SPINS FIRST

We take a spins-first approach to quantum mechanics. While several prior textbooks also take this approach, most are for more advanced students [4-7]. Our own spins-first textbook, tailored for the middle/upper-division audience, is currently in press [8]. We are able to make this approach less abstract by using an explicit computer simulation of Stern-Gerlach (SG) experiments, originally developed by Schroeder and Moore [9].

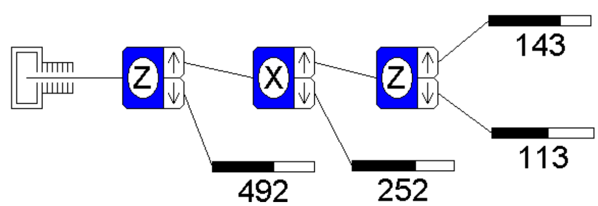

FIGURE 1. A java simulation of successive Stern-Gerlach measurements is a concrete representation of the quantum measurement process.

Our story begins with exactly that, a story—which is a very powerful and memorable representation. We discuss the (fictitious) story of Erwin Schrödinger sorting his socks—black vs. brown, long vs. short. By making an explicit analogy with this everyday occupation and successive SG measurements shown in Figure 1 (sock color corresponds to S_z, sock length corresponds to S_x), students are able to encounter from the first day of class the counterintuitive nature of quantum mechanics. In problem-solving interviews, one student mentioned that he was still trying to make sense of the sock story. We learned from his remark that stories can be a vivid way of recording one's lack of understanding as well as one's understanding.

DIRAC BRA-KET NOTATION

It is straightforward and natural to use Dirac bra-ket notation in our spins-first introduction to quantum mechanics. In bra-ket language, the mathematical representations of states $|\psi\rangle$, projections $\langle +|\psi\rangle$, observables $\hat{H}$, and projection operators $|+\rangle\langle +|$ are all visibly different from each other. It is relatively easy to draw students' attention to the physical differences among the properties that these symbols represent. We found in the first year that we taught using bra-ket notation that many students spontaneously and appropriately used this notation in a later paradigm on the hydrogen atom; encouraging us to adopt this notation throughout our quantum courses and eventually encouraging us to put spins before even a basic introduction to wave mechanics in the form of particle-in-a-box. We also see students' use this notation by preference in problem-solving interviews [5].

From the very beginning of the course, we also emphasize the postulates of quantum mechanics. Successive SG measurements are exploited to represent concrete examples of the postulates. Our students rapidly come to understand that (scalar) inner-products are the answers to most of the questions they are asked. They can reliably calculate these inner products, especially in bra-ket notation. However, when we later introduce the projection postulate, students often do not realize that this second use of the word "projection" includes the "direction" in abstract Hilbert space. We are actively developing an activity involving degeneracy that seems to help at least some students through this representational confusion.

MATRIX REPRESENTATIONS

$$\begin{array}{c|cc} A & |+\rangle & |-\rangle \\ \hline \langle +| & \langle +|A|+\rangle & \langle +|A|-\rangle \\ \langle -| & \langle -|A|+\rangle & \langle -|A|-\rangle \end{array} \tag{1}$$

Because our students are (basically) comfortable with bra-ket notation, it is natural for us to use this notation to describe matrix elements as well. Equation (1) shows the matrix for the operator A in the z-basis of a spin ½ system. Notice that we explicitly label the rows and columns with the appropriate basis elements. Similarly, we find it useful to explicitly label other symbols when we first introduce them, e.g. Figure 2.

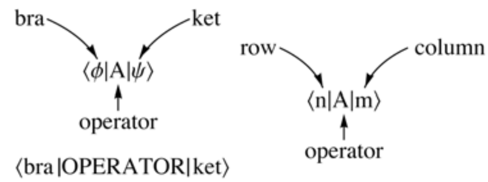

FIGURE 2. Taking time to explicitly label representations is helpful to many students.

While this matrix notation has many advantages, particularly that it makes the basis explicit; we have seen disadvantages in a later course in classical mechanics, when students were introduced to the alternative notation A_{ij} for the matrix elements. Because matrix multiplication is assume in bra-ket notation, our students clearly had never thought about this notation as summation and had trouble transferring their understanding to the alternative notation.

WAVE FUNCTIONS

In several activities and especially in homework problems, we ask students to make histograms of the probability distribution of some quantum experiments (Figure 3).

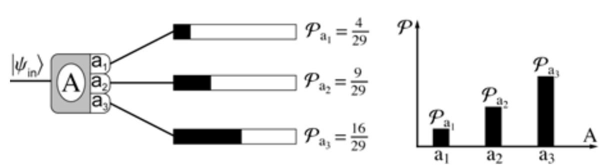

FIGURE 3. Histograms emphasize that quantum measurements are always probabilities.

When we later introduce wave functions, it is natural to describe the wave function as the projection of the state onto a position eigenstate as shown in equation (2). (We do not discuss the subtle issues about dimensions unless students bring this up.)

$$|\psi\rangle \doteq \begin{pmatrix} \langle x_1|\psi\rangle \\ \langle x_2|\psi\rangle \\ \langle x_3|\psi\rangle \\ \vdots \end{pmatrix} \begin{array}{l} \leftarrow x_1 \\ \leftarrow x_2 \\ \leftarrow x_3 \\ \quad\vdots \end{array} \tag{2}$$

The square of the norm of the wave function then becomes a graphical representation of the continuous limit of the discrete histograms that the students have been examining for some weeks (Figure 4).

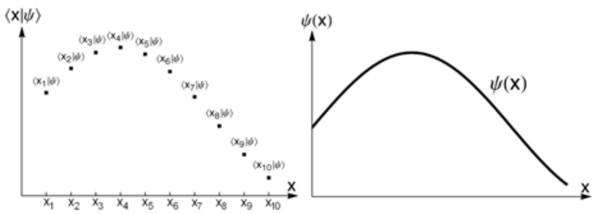

FIGURE 4. These representations for a wave function emphasize that it $\psi(x)$ is the projection of the state onto a position eigenstate.

Beginning with our E&M paradigms, we often use color to represent the value of a scalar field, particularly in three spatial dimensions. This representation has implications for how we represent the distribution of the probability density in space. Figure 5 shows three representations for the probability density of a particle confined to a sphere (i.e. the rigid rotor problem) [11].

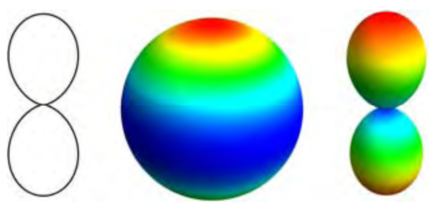

FIGURE 5. Multiple representations for the probability density for a spherical harmonic, i.e. a possible state for a rigid rotor. What does the radial direction mean?

Of course, by plotting only probabilities, we have been ignoring an important point that causes endless confusion for our students. The coefficient of a quantum state in a particular basis direction (and therefore a quantum wave function) is inherently complex. Only the probability, which is the square of the norm of this coefficient, is real. Even after many weeks of emphasis, a few students will still confuse $\langle x|\psi\rangle$, $\langle x|\psi\rangle^2$ and $|\langle x|\psi\rangle|^2$. This problem has been exacerbated in recent years as complex numbers disappear from high school and university level mathematics curricula. A delightful activity intended to help students focus on the complex-valued nature of the wave function itself is to have them represent the (complex) values of spherical harmonics as many little complex planes on the surface of a balloon, as shown in Figure 6.

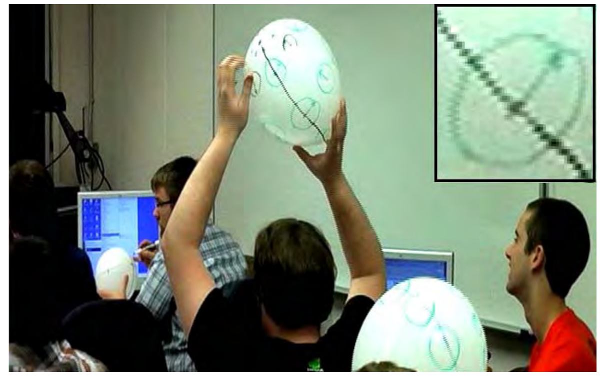

FIGURE 6. Students draw the complex phases for spherical harmonics on the surface of balloons.

TIME DEPENDENCE

We also employ the different representations for the spatial dependence of the probability density in animations that depict time dependence. Figure 7 shows a still picture of an animated gif [12].

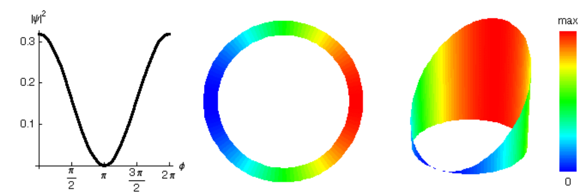

FIGURE 7. Representations for the probability density of a quantum particle confined to a ring.

There are times when having the students represent a physical situation with their own bodies helps them reason through the geometry. These "kinesthetic" activities are among the most memorable for students, who often refer to them in other classes or in our senior exit interviews. We find them to be particularly effective in situations involving three dimensions in space and/or situations involving time dependence. Figure 8 shows students acting out the time dependence of an envelope function in the Periodic System paradigm.

FIGURE 8. Students act out an oscillating envelop wave function in a course on Periodic Systems.

CONCLUSIONS

When writing this article, we ourselves are struck by how many representations we ask students to wrap their brains around. We believe that the extra representations that we use are sense-making tools for the students. By spending time on these various representations, students also have time to ask questions about them (of themselves and of us) and to build their own robust understanding. We believe that we see evidence in class and in problem-solving interviews of students making use of these representations. For example, we hear students asking what an operator "is" and drawing Stern-Gerlach devices to organize their thinking.

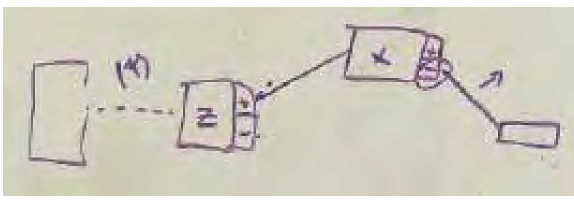

FIGURE 9. A student draws SG devices to organize his thinking during a problem-solving interview.

However, we also hear students expressing confusion related to these representations. For example, several students mentioned still struggling with visualization and geometric reasoning. Students frequently conflate the eigenvalue equation with the SG devices [10]. Clearly, there is lots of work remaining, both in terms of curriculum development and in terms of research on student understanding of quantum mechanics.

ACKNOWLEDGMENTS

We would like to thank all of the Paradigms students and TA's over the years who have contributed so much to our understanding. We would also like to thank Kerry Browne for his insights and programming regarding the use of color to represent probabilities and Ethan Minot for his delightful balloon and kinesthetic activities. Our thanks to Jeff Monroe, Teal Pershing, and Andy Svesko for locating many of our examples. This work was also supported by NSF grant DUE 0618877.

REFERENCES

1. C. Singh, M. Belloni, and W. Christian, *Physics Today*, **59**, 43 (2006).
2. C. Singh, *American Journal of Physics*, **69**, 885 (2001).
3. Further information about the Paradigms Project, including all of the activities discussed here, can be found at: http://physics.oregonstate.edu/portfolioswiki
4. Feynman, Leighton, and Sands, *The Feynman Lectures on Physics, Volume 3, Quantum Mechanics*, Reading, MA: Addison-Wesley, 1965.
5. Cohen-Tannoudji, Diu, and Laloe, *Quantum Mechanics*, New York: Wiley, 1977.
6. Sakurai, *Modern Quantum Mechanics*, Addison-Wesley, 1985.
7. J.Townsend, *A Modern Approach to Quantum Mechanics*, University Science Books, 1992.
8. D.McIntyre, *Paradigms in Physics: Quantum Mechanics*, San Francisco: Pearson Addison Wesley, in press.
9. The original version of this simulation is reported in: D. V. Schroeder and T. A. Moore, *Am. J. Phys.*, **61**, 798-805 (1993); subsequently the simulation was ported to java and is now available from the Open Source Physics Project: http://www.opensourcephysics.org/
10. E. Gire and C. A. Manogue, 2008 PERC Proceedings (2008); E. Gire and C. Manogue, submitted to 2011 PERC.
11. http://physics.oregonstate.edu/portfolioswiki/acts/cfqmsphere
12. http://physics.oregonstate.edu/portfolioswiki/acts/cfqmring