Concepts of derivatives

What is a derivative?

1. A ratio of small changes. This is the limit definition of a derivative, and seems to be the first learned by students, and the first forgotten.

2. A measurement to determine its value. This involves measuring small changes and taking a ratio.

3. The slope of the tangent to a curve

4. The result of manipulation of a symbolic expression. For many students, this is their primary understanding of “finding a derivative.”

What is a partial derivative?

1.2. When considering a ratio of small changes or a measurement, a partial derivative requires that we ask not only which quantities are changing, but also which quantities to hold fixed.

3. A tangent line turns to a tangent plane in two dimensions, and a partial derivative becomes the slope of the plane in a given direction at a given point.

4. The procedure to find a partial derivative of a symbolic expression is identical to that for an ordinary derivative, provided there are not interdependencies among the variables in the expression. It is unsurprising that many of our students believe that a partial derivative means "everything else is held fixed."

Instructions to build a PDM:

http://www.physics.oregonstate.edu/portfolioswiki/whitepapers/pdm:start

Experts' understanding of partial derivatives using the Partial Derivative Machine

Experts interviewed

Physicists
One associate professor and one full professor. Both use computational methods in their research in astrophysics and optics.

Engineers
One chemical engineer who is a full professor with considerable research and teaching expertise in thermodynamics, and two engineers who study student thinking and epistemology in engineering.

Mathematicians
Two assistant professors whose research is in mathematics education at the collegiate level.

Measuring the derivative

Physicists and engineers
Immediately (within two minutes of taking data) computed a "ratio of small changes" value for the derivative. Soon verified that the changes were small enough.

Mathematicians
Required 30 minutes and four prompts from the interviewer before finally recognizing that they could develop an experimental measurement for the derivative. Repeatedly returned to speculation regarding the functional form of $x$, even after being prompted to find a numeric value.

What about $y$ and $F_y$?

Physicists
Assumed that $F_y$ must be held fixed, rather than $y$ when taking the derivative. Recognized that only two variables were independent and that one of these could be $y$.

Engineers
Similar to physicists, in assuming that $F_y$ should be held fixed. Two engineers had confusion in the number of independent variables.

Mathematicians
Based on physical gestures, recognized that the derivative would differ if $y$ were held fixed. Did not discuss independent variables.