Notation

1 Vectors

As already noted, we *never* write vectors as pairs or triples of numbers; this notation is reserved for coordinates, a quite different concept. The symbols we use for vectors have arrows on them (to match what we write by hand) as well as being bold-faced (to match the notation usually used in textbooks). The one exception to this rule is that we put hats on unit vectors, rather than arrows. [1]

2 Spherical Coordinates [2]

2.1 The Problem

Nearly everybody uses r and θ to denote polar coordinates. Most American calculus texts also utilize θ in spherical coordinates for the angle in the equatorial plane (the *azimuth* or *longitude*), ϕ for the angle from the positive z-axis (the *zenith* or *colatitude*), and ρ for the radial coordinate. Virtually all other scientists and engineers — as well as mathematicians in many other countries — reverse the roles of θ and ϕ (and use some other letter, such as R, for the radial coordinate).

Why is this a problem? After all, the change in notation only affects students in particular fields, such as physics or electrical engineering. Furthermore, it's just a convention; surely these students have the maturity to

[1] To our dismay, some journals seem to have difficulty typesetting hats above bold math italic Greek letters, such as $\hat{\boldsymbol{\theta}}$.

[2] This section is adapted from [1].

deal with it. Based on our experience trying to implement this change during a second-year course in multivariable calculus, we feel that such sentiments underestimate the extent of the problem. Students find the complete interchange of the roles of θ and ϕ to be terribly confusing — and once confused, always confused.

Using different names for the radial coordinate, on the other hand, causes few problems. The use of r for the spherical radial coordinate can be confused with the radial coordinate in polar or cylindrical coordinates, but computations requiring both at the same time are rare. While ρ is not available to the physicist, as it is used to represent charge or mass density, students do not appear to be confused by the use of several different names for the spherical radial coordinate.

There is however a much more serious problem. Several of the most commonly used calculus texts list spherical coordinates in the order $(\rho,\ \theta,\ \phi)$; the rest use $(\rho,\ \phi,\ \theta)$. The first of these is left-handed! An orthogonal coordinate system is right-handed if the cross product of the first two coordinate directions points in the third coordinate direction. This is immaterial in the traditional mathematics treatment of vector calculus, but crucial to the way physicists and engineers treat the same material. These scientists often introduce basis vectors in the coordinate directions, analogous to $\{\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{k}}\}$ for rectangular coordinates, and it is essential that these vectors form a right-handed system. This requires that the zenith be listed before the azimuth; with the standard mathematics convention, this is $(\rho,\ \phi,\ \theta)$. Books which use the standard mathematics definitions of the angles but write $(\rho,\ \theta,\ \phi)$ are doing their students a major disservice, although we reiterate that this is only an issue for material covered in subsequent courses.

2.2 The Solution

There is a uniform standard for the use of spherical coordinates in applications, which is nowhere more apparent than in the definition of spherical harmonics. These special functions on the sphere are widely used, notably in the quantum mechanical description of electron orbitals, which in turn underlies much of chemistry. It can not be stated too strongly that *everyone* writes the spherical harmonics as $Y_{\ell m}(\theta, \phi)$, where θ is the zenith and ϕ the azimuth. There is simply no way to change this convention, which is embedded in generations of standard reference books.

One objection to this is that it is confusing to use the same label, θ, for

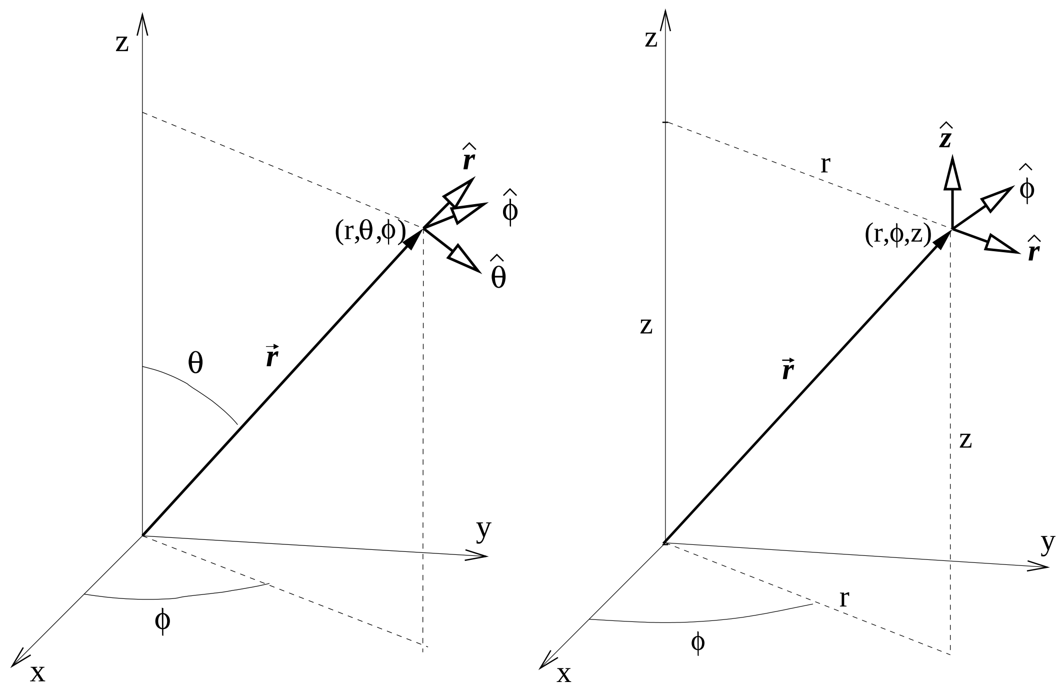

Figure .1: Our conventions for spherical and cylindrical coordinates.

two different angles in polar and spherical coordinates. This objection can be easily resolved, even if the resolution may not be popular: Change the conventions for polar coordinates, that is, use ϕ rather than θ.

We propose that these conventions be adopted by mathematicians, and we use them throughout these materials. Our conventions for both spherical and cylindrical coordinates are shown in Figure .1.

3 Integrals

There are two common notations for multiple integrals, one being to use one integral sign for each iterated integral which will ultimately be performed, the other is to use a single integral sign for each integral, since an integral just means "add things up". We use the latter notation for surface and volume integrals, but iterated integrals are written out in full. For instance, when finding the flux of $\hat{\boldsymbol{k}}$ upwards through the unit disk, we write

$$\int\limits_D \hat{\boldsymbol{k}} \cdot d\vec{\boldsymbol{A}} = \int\limits_0^{2\pi} \int\limits_0^1 1\, r\, dr\, d\phi \tag{1}$$

The in-between case in which the limits are not given explicitly can be written either way depending on the context (and personal preference), as in

$$\int\limits_D r\, dr\, d\phi = \iint\limits_D r\, dr\, d\phi \tag{2}$$

Bibliography

[1] Tevian Dray and Corinne A. Manogue, *Spherical Coordinates*, College Math. J. **34**, 168–169 (2003).

[2] Robert Osserman, **Two-Dimensional Calculus**, Harcourt, Brace, and World, New York, 1968.

[3] Tevian Dray & Corinne A. Manogue, *The Murder Mystery Method for Determining Whether a Vector Field is Conservative*, College Math. J. **34**, 238–241 (2003).