The Integration of Interactive Activities into Lecture in Upper Division Physics Theory Courses

by

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Introduction

Research has shown that the traditional style of teaching physics, students passively listening and taking notes, is proving itself rather ineffective (1,2,3,4,5). This finding has led many institutions to reform their lower-division physics programs in an effort to enhance students' learning. Oregon State University, has chosen to reform it's upper-division curriculum through a program called "Paradigms in Physics."

The main goals of the "Paradigms in Physics" reform are the following (6):

- "To improve students comprehension by cultivating their analytical and problem solving skills."
- "To provide bridges between the content of different sub-disciplines of physics."
- "To give the students a thorough understanding of a set of important examples 'paradigms' which underlie the professional physicist's view of more complex problems."
- "To make use of alternative teaching strategies such as group projects and discussions, group problem solving sessions, regular labs, computer labs, spread sheet and worksheet computations, individual or group presentations of individual investigations, etc."

In working to attain these goals, the professors teaching the Paradigms have developed and implemented classroom activities utilizing a variety of teaching techniques alternative to traditional lecture.

This research project is a preliminary qualitative analysis of some of the activities implemented in Preface 1 and Preface 2 of the Paradigms in Physics reform. The first two quarters of the reform begin with a weeklong preface. The preface is designed to get all the students up to speed by introducing the basics of some concepts needed for the paradigms to follow. The study will develop some hypotheses for further study about when, where and how is it effective to integrate interactive activities into lecture in upper division physics theory classes.

The activities studied in this research project were developed to make the classroom environment more student-centered rather than teacher-centered. The researcher's study of literature and research on education in physics and other disciplines, in addition to the professor's expertise in teaching physics, were important in the development of the activities used in this study. The paragraphs below describe a few educational studies that were particularly significant in the development of activities that involve the instructional methods of student collaboration and in class worksheets.

Educational studies have shown that cooperative group work enhances students' conceptual understanding by allowing them 1) to articulate their present thoughts, b) to reflect on their own ideas and those of others, c) to elaborate on their thoughts and d) to evaluate multiple ways in which a question may be answered. Use of this type of cooperative approach allows all students to discuss their own ideas or questions regardless of whether they are comfortable presenting their ideas to the entire class (10).

In recent years, collaborative learning has been shown effective in teaching lower-division physics. Eric Mazur, a pioneer in physics education research, has been teaching introductory physics at Harvard University using a method of student collaboration referred to as *Peer Instruction* (4). With the method of *Peer Instruction*, lectures consist of a number of short presentations on key points, each followed by a concept test consisting of short conceptual questions on the subject under discussion. Students first answer the concept test individually and record their answer, then they are asked to try to convince their neighbor why their answers are correct. In his book entitled *Peer Instruction*, Mazur describes how he has implemented this teaching technique in his large lower-division physics lectures and reveals the research results showing the effectiveness of this method over that of traditional lecture (4).

Literature on lower-division physics education has shown that in-class worksheets are another useful strategy for increasing student interaction and understanding. Alan Van Heuvelen, a physics professor at New Mexico State University, has developed a curriculum based upon a set of worksheets called *Active Learning Problem Sheets* (the ALPS kit). Students in his class would participate in answering questions and solving problems on the sheets through interaction with their neighboring students. He used the sheets in several ways. Sometimes they aided the students in constructing concepts by systematic analysis of a series of thought experiments, while other sheets work on the development of specific skills such as constructing a free body diagram. Still other sheets addressed misconceptions and helped students to develop problem-solving skills. The sheets were integrated into lecture by allowing the students to work alone and then asking them to compare their answer with their neighbors, making an attempt to reconcile any differences in their answers (8). Van Heuvelen's techniques seemed to receive positive feedback although no formal assessment has been done.

As a final example, The Physics Education Group, directed by Lillian McDermott at the University of Washington, has developed a set of instructional materials that supplement the lectures and textbook of a standard introductory physics course. These

"Tutorials in Introductory Physics" are similar to a series of worksheets and emphasize the development of important physical concepts and scientific reasoning skills rather than solving standard quantitative problems found in traditional textbooks. During a tutorial session, students complete sequenced tasks and answer questions. They construct answers through discussions with their classmates and with tutorial instructors. The "Tutorials in Introductory Physics" have significantly improved students understanding of concepts in introductory physics over the traditional lecture style of instruction (9).

Although the majority of physics education research has been done at the lower-division level, it is likely that many of the successful ideas and techniques can also be used to teach upper-division physics. The six activities used in this study were developed with this in mind and combine aspects from many of the techniques mentioned above. The professor who directed the development of all the activities has over 10 years of experience teaching physics. Her insight into what areas of physics frequently cause students difficulty was helpful in deciding where in the curriculum an interactive activity might improve student learning.

Each activity was developed to achieve specific instructional goals and objectives. Some activities served to review a technique and/or concept, others introduced a new concept and/or technique, while others provided students with examples. The structure of the activities varied, although most of them included in-class worksheets completed in small groups. Some worksheets used guiding questions to lead the students to realize a concept while others emphasized problem solving by asking a question and requiring students to develop a strategy for solving the problem. A detailed description of each activity is included in the profile for that activity.

Method

This research project is a preliminary study conducted in a qualitative manner. It was not possible to do a quantitative study for two reasons. First, the Paradigms in Physics curriculum reform is a recent development with a limited number of students in the curriculum. The sample population that completed the activities under analysis consisted of 16-18 junior year physics and engineering-physics majors. Second, while there are formal assessment instruments such as the Force Concept Inventory (5) available for lower-division physics, there is not a comparable assessment accepted for use in upper-division physics.

The researcher choose to study activities implemented in the week-long Preface 1 and Preface 2 of the Paradigms in Physics curriculum for the following reasons: all activities were being taught by the same instructor, the activities constituted a coherent set of activities on a few topics, the content of each activity was similar in that it came from the math department but was being given application to physics, and all of the activities involved either small groups and/or a classroom discussion.

The first stage of the project involved the researcher collecting data on the implementation of each activity in the form of feedback from various sources. The majority of the data used in the study were accumulated during and in the few weeks following Preface 1 and Preface 2. However, a small portion of the data used in the "General Observations Pertinent to all Profiles" and "Procedural Advice" sections were collected throughout Fall term 1997 and Winter term 1998. Data was collected in the form of interviews, surveys, course notes, student worksheets and homework, and observation.

The sources of data are listed below and the means by which the data were obtained is contained in the parenthesis after each data source.

- the observers:

  Katherine S. Meyer, Researcher and Doctoral student, Department of Physics, Oregon State University

  Adam Wolfer, Doctoral student, Science and Mathematics Education Department, Oregon State University; (observation notes and discussion)

  Tobias Moleski, Masters student, Department of Physics, Oregon State University (observation notes and discussion)

- the students in the Preface 1 and Preface 2 courses (survey questionnaires, formal interviews, worksheets, homework, classroom performance)
- the course instructor of Preface 1 and Preface 2, Corinne A. Manogue (course notes, journal, interviews and discussions)
- the course teaching assistants; Preface 1: Kerry Brown; Preface 2: Jason Janesky (interviews and discussions)

The instructor identified the goals and objectives of each activity. An instructional goal describes, on a global level, something that students are to learn and is not easily assessed. An instructional objective describes specific skills or competencies that a student should gain from instruction. Objectives can be assessed. Profile objectives may be any of the following three types:

- Review Objective(s) – designed to provide students with a review of past days material

- Primary Objective(s) – the main objective for that day
- Preview Objective(s) – designed to give students a preview of the coming material

A profile was written for each activity, which contains the following: the instructional objectives for the activity, the planned implementation strategies, the actual implementation, and an assessment of the objectives. Usually comparisons between different data sources were used to achieve the assessment of the objectives. More details about the information contained in each profile can be found in the "Profile Guidelines" section.

Finally, the method of triangulation was used to analyze the profiles for each activity. Triangulation is the process of using a variety of data sources in a study to verify the analysis (10). The researcher compared the profiles, finding commonalties and differences between activities and noting patterns that began to emerge between them. The researcher had the following questions in mind during the investigation of the activities. Are there collaborative activities that work well and how might they be structured? What size groups are optimum? What balance should there be between worksheet time and lecture time? What makes worksheets most effective? What characteristics of an in-class activity lead to that activity being more successful? What characteristics of an in-class activity lead to that activity being less successful? The information obtained from this analysis allowed the researcher to develop a list of characteristics and implementation strategies of effective and in-effective activities. The researcher then refined and generalized the list into a set of hypotheses for further study.

Profile Guidelines

I. Objectives

A. Objectives

1. What are the instructor's objectives for the activity? (e.g. Review, Primary, Preview)

II. Planned Implementation Strategies

A. What and When Developed

1. What day did activity begin and end? (e.g. Began Day 4 of Preface 1 [1 hr & 50 min. class], ended Day 5 of Preface 1 [50 min. class])
2. What did activity consist of? (e.g. brief lecture on ____ followed by worksheet, followed by closure lecture on worksheet)
3. Who developed activity? Is activity self-explanatory or does it require class involvement, etc.? If there was a worksheet, then who developed it?
4. How long was activity or worksheet expected to take for completion?

B. Laying of Groundwork for Activity

1. When, if at all, was the groundwork laid for the activity? What was this groundwork? (e.g. Homework the night before, or lecture the day before etc.)
2. How did this groundwork (e.g. students learned methods to do ____) get tied into worksheet or activity (e.g. students applied methods to specific problems in worksheet.)

C. Structure of Class During Activity (to include Instructor, TA, and students)

1. How would students be grouped, if at all, for activity? (e.g. randomly grouped in 3's or individually) What would be given to students in way of materials? (e.g. each student would receive his/her own copy)
2. If there is a worksheet, how is it incorporated into the activity? (e.g. students completed worksheet and volunteers came to board and plotted their results.)
3. What are instructor and TA doing while groups complete worksheet/activity (e.g. walking around asking or answering questions)?
4. How are they checking that objective is met?

D. Closure to Activity

1. What will bring closure to the activity? (e.g. class discussion, instructor lecture summarizing important points, or concept under investigation might be realized)
2. What questions will be asked of the students?
3. What would happen after this activity? (e.g. cover new material)

III. Actual Implementation and Assessment

A. Did the overall activity go according to plan? (yes/no)

B. Description of how the actual implementation differed from the planned implementation.

1. A description of any unanticipated problems and/or benefits of the activity.

2. The researcher will use information obtained from various data sources to analyze the positive and negative aspects of the activity.

Some examples of likely topics to be discussed here are:

- Length of the activity
- Group dynamics during the activity
- Group presentation of results and/or class discussion of results, to include realization of concept under investigation (if applicable)

4. Generalizations about how students responded to the activity based upon feedback from all of the data sources. Relevant comments from the students or instructor included either as direct quotes or paraphrased.

C. Assessment of Objectives

1. Overall did the activity meet most of the objectives?

Each objective will be addressed and a description of the evidence of its being met will be provided.

The Study of Complex Numbers

A significant portion of Preface 1 involved the study of complex numbers. The following activities were developed to investigate this topic on Day 2 and Day 3 of Preface 1: *The Complex Plane*, *Phases I*, and *Phases II*. The instructor had the following instructional goal for these two days:

Goal: Students would understand θ , $e^{i\theta}$ and $re^{i\theta}$ geometrically, algebraically and graphically.

Profile: *The Complex Plane* Activity

Objectives

The instructor's objectives for the activity were as follows:

- **Review Objective:** Given a complex number the students will be able to recognize a complex number as a pair of real numbers and will be able to plot it on an Argand diagram, using polar coordinates if desired.
- **Primary Objectives:**
  A. Given a set of complex numbers in the form $e^{i\theta}$ the students will be able to plot these numbers on an Argand diagram and discover that θ is the angle from the positive x-axis and that all points are unit distance from the origin.
  B. Given a set of complex numbers in the form $e^{i\theta}$ the students will be able to recognize that the distance from the origin to a point is the norm of the complex number represented by the point in the Argand diagram.
  C. Given important special cases such as 1, -1, i, and $-i$ the students will be able to write these values in exponential form. Given the values in exponential form, the students will be able to write them in rectangular form.
- **Preview Objective:** Given a set of complex numbers in the form $re^{i\theta}$ the students will be able to hypothesize that r is the distance from the origin.

Planned Implementation Strategies

The Complex Plane activity was planned to begin and end on Day 3 of Preface 1 (50-minute class). The activity would consist of a brief lecture given by the instructor followed by students' completion of the worksheet, an instructor wrap-up of the main concepts in the worksheet and 5 minutes of lecture on the norm and phase of complex numbers. The self-explanatory worksheet, developed by the instructor with assistance from the graduate student researcher, was intended to guide the students toward the instructor's primary objectives for the activity. According to the developers, *The Complex Plane* worksheet would take a group of three average junior year physics students about 15 minutes to complete. The activity was transitional in nature because it began with techniques familiar to the students then progressed to investigate new material.

The groundwork for the activity would be laid during lecture the previous day. The instructor lectured on the complex plane and Euler's formula, teaching the students methods for calculating the rectangular form of a complex number, how to plot complex numbers on the complex plane, and how to calculate the norm of a complex number. The students applied these methods to specific problems during *The Complex Plane* activity.

The students would be grouped randomly in threes; in a class of 18 students, six groups would be formed. The instructor would group students to reduce the number of classroom neighbors placed in the same group. Groups would then be directed to a position in the classroom corresponding to their group number where they could arrange their individual, mobile desks facing one another in a circle. Each student would receive his/her own copy of the worksheet and a piece of polar graph paper. Calculators would not be allowed.

While groups completed the worksheet the instructor and teaching assistant (TA) would observe and periodically ask and answer students' questions. More specifically, the TA and instructor would listen to group discussions looking for evidence that they understood the geometric significance of θ and at the appropriate time ask groups to explain the geometric significance of θ.

The closure to the activity would summarize the important points of the worksheet, such as the unit circle, and ask questions of the whole class such as: What is $e^{i\pi}, e^{i\pi/2}$? What is another name for: 1, -1, i? Following the closure, a 5-minute lecture would be presented on the exponential form for complex numbers emphasizing the geometric meaning of r in the expression $re^{i\theta}$. After this activity the instructor would begin an activity on phase transformations (see *Phases I* and *Phases II*).

Actual Implementation and Assessment

Overall, the implementation of the activity did not go according to plan. The main difficulty was that the completion of the worksheet took about 45 minutes. As a result, the activity was not completed on Day 3 and the wrap-up had to be extended to Day 4 of Preface 1. Some factors that may have created this situation are listed below. The length and difficulty confused many students. Students were not able to link together all of the sections of the worksheet and even those familiar with the material seemed to lose the big picture. The main concepts of the worksheet were not obvious since they were broken up into many questions of seemingly equal importance.

When estimating the time to allow for the worksheet portion of the activity, the instructor and researcher had not considered that portions of the worksheet might be troublesome and require additional time. They quickly realized during the activity, when several groups had questions, that each group's questions could not be answered efficiently on an individual basis. As a result some groups skipped over troublesome portions of the worksheet and moved on to the next question, causing them to get out of sync with the remainder of the class and disrupting the flow of the activity. As students completed the worksheet, the instructor recognized it was necessary to regroup the class at certain points to ask questions that check student understanding of the main point. One student appreciated these stopping points and remarked that he liked being pulled back together with the class while completing the worksheets.

The first question on the worksheet caused the majority of students' problems. Most groups spent 15 minutes on it, which was the allotted time for the entire worksheet. They had difficulty remembering whether cosine was an odd or even function, and how to calculate the cosine and sine of $\pi / 6$ and negative angles. The second question on the worksheet also took longer than expected because, to the instructor's surprise, students had trouble plotting complex numbers on polar graph paper. Many students seemed unfamiliar with polar graph paper and expressed relief when given rectangular paper, as they had not reached the level of understanding required to apply their knowledge of complex numbers and the complex plane to the process at hand. In both of the above cases students did not have the background knowledge expected of them.

Many students did not understand how they were to interact with their group members when completing the worksheet. This problem was evident from several students' worksheets that had the correct answer to a question but did not show any intermediate steps, leading the researcher to believe they did not think through the problem themselves. In an interview after Preface 1, one student remark showed that a few groups did in fact work well together, discussing the worksheet amongst themselves as they progressed. This student best liked working in groups and explaining things to others.

Many students had difficulty with question 3 of the worksheet and when questioned by the TA or instructor, found it challenging to explain the geometric significance of θ. Surprisingly, the group that struggled with the worksheet the least was not an exception. Further questioning of this group, toward the end of the activity, showed they did not have a thorough understanding of the graphical interpretation of θ .

As students approached question 6, the instructor and TA walked around and looked to see if they knew they were plotting the "unit circle." All groups had difficulty recognizing they were plotting points on the unit circle and over half the students were unfamiliar with the term "unit circle." The group that struggled the least when completing the worksheet realized they were plotting a circle and that it was the "unit circle" but were confused by the complex plane.

After spending twice as long as expected on the worksheet, the majority of groups were almost to question 6 when the instructor interrupted, pulling the entire class together for the last 10 minutes for closure. She explained the solutions to questions 1 through 5, (making sure they saw the geometric significance of θ) and proceeded to question 6 and 7 and closed with asking the entire class: "What is $e^{i\pi}, e^{i\pi/2}$ and what is another name for: 1, -1, i?" Most students had to finish questions 7 through 13 for homework.

Additional closure of the activity was done through lecture at the beginning of class on the following day (Day 4).

The instructor wrap-up, summarizing what students should have learned from the worksheet, was seen as valuable in the eyes of all students. Although it was accidental, having the wrap-up of the worksheet stretch over to the next day was viewed favorably by the students. Many of them mentioned that having time to study and think about the material before seeing it for a second time increases their ability to recall new concepts and techniques. One student commented, "…it was nice to know I got it right," while another wrote,

> Rarely do I ever have an epiphany or insight during lecture. It always occurs when I work on it alone. If it is a difficult concept then after some solitary effort a little one-on-one instruction helps tremendously. The worksheets help guide during the solitary effort. They keep me from total mental oblivion.

Reflecting on Preface 1, the instructor realized she might have tried to accomplish too much with the activity. Her attempt to give a brief review, meet the main objective, give a brief preview of what was to come and give examples through the activity may have caused students to miss the main point. In light of this, it is interesting to note that one student who did not feel the wrap-up was a summary of what was learned from the worksheet, commented that it was "mostly new material."

Despite several challenges, the activity might have increased student learning over methods used to teach the same material in past years. According to the instructor, learning was happening sooner and in more depth than in past years of teaching Mathematical Methods for Physicists. The material previously taught in the Mathematical Methods for Physicists course was now spread throughout the paradigms courses. The instructor was impressed with a student question asked during office hours immediately following the activity, and remarked that the question was asked unusually early. The student inquired about the difference between $e^{i\theta}$, which traces out a circle, and $e^{-anything}$, which decreases exponentially for increasing "anything". He said, "You could have hit me over the head with a brick yesterday (Tuesday) in class and I wouldn't have understood this", but after the instructor explained it the next day, he seemed to grasp it.

It was clear that *The Complex Plane* activity did not go as planned and as a result did not meet the instructor's objectives in the planned way. However, when the instructor re-grouped the class during the completion of the worksheet, to ask questions drawing out the main points, it was evident that the majority of the class had met the instructor's primary objectives A and B. The following day students successfully plotted their complex numbers (in exponential form) on an Argand diagram during the *Phases I* worksheet, making it obvious that the review and preview objectives from *The Complex Plane* activity had been met. One student's performance at the front board during the *Phases II* activity showed evidence that objective C had been partially met, however it was not evident whether the entire class had met this objective as it was not directly assessed. In addition, all students felt the worksheet portion of the activity deepened their understanding of the methods they had learned the day before (Day 2) and one student mentioned the worksheet was useful as a reference.

NAME ____________________

Preface 1 The Complex Plane

1) Find the rectangular form for each of the following complex numbers:

$$z_1 = e^{\frac{i\pi}{2}}, \qquad z_2 = e^{i\pi}, \qquad z_3 = e^{-\frac{3i\pi}{4}}, \qquad z_4 = e^{-\frac{i\pi}{6}}$$

Notice that all of these points are in the form $z = e^{i\theta}$ for real values of θ.

2) Plot the values of z_i from question 1) in the complex plane, on the graph paper provided.

3) What is the geomentric significance of the value of θ? What units are you using?

4) How far is each point from the origin?

5) Choose a value of θ on your own and plot your z_5 on the same graph.

6) Consider the set of points $e^{i\theta}$ for $0 < \theta < 2\pi$. Would these points trace out a geometric shape in the complex plane? If so, what shape?

7) Keeping in mind your answers to 5) and 6), what might you call this shape? Check with the instructor for the real name.

8) Calculate the norm of $e^{\frac{i\pi}{9}}$.

9) Calculate the norm of $e^{i\theta}$.

10) If you change θ does the norm change?

11) Is this norm what you would expect, given your answers above? Explain.

12) Plot $z_6 = 3z_1$ and $z_7 = 5z_4$ on the same graph as before.

13) How far are z_6 and z_7 from the origin? Calculate the norms of z_6 and z_7. Compare the distance from the origin with the norm.

Profile: *Phases I* Activity

Objective

The instructor had the following objective for the *Phases I* activity:

- **Primary Objective:** Given a complex number, z_1, the student will be able to recognize that multiplying z_1 by $e^{i\delta}$ for a specific angle δ rotates z_1 in the complex plane by the angle δ.

Planned Implementation Strategies

The *Phases I* activity was planned to begin and end on Day 4 of Preface 1 (1 hour and 50 minute class). The activity would consist of the following: students' individual completion of the *Phases I* worksheet (developed by the instructor with assistance from the graduate student researcher and teaching assistant), designated students plotting on the complex plane on the front board, and an instructor-guided class discussion of these plots. After the self-explanatory worksheet was completed, students would be prepared to discuss, analyze, and draw conclusions from the graphs they had constructed in the worksheet. The Phases I activity would be implemented in the computer lab and was estimated to take about 15 minutes. The activity was transitional in nature because it began with techniques familiar to the students, then progressed to investigate new material.

The groundwork for the *Phases I* activity was laid earlier in the week, especially through *The Complex Plane* activity. Students were expected to be familiar with the following methods that were required to complete the worksheet:

- Algebraic and arithmetic manipulation of complex numbers in the exponential form.

- Plotting of complex numbers in the complex plane.

The students would be counted off by threes and assigned a value of the complex number z_1, corresponding to their number ($z_1 = 2e^{i\pi/2}$, $3e^{i5\pi/4}$ *or* $4e^{-i\pi/3}$). Students would complete their worksheets individually at the stationary desks in the computer lab. The TA would give each student a worksheet and a piece of rectangular graph paper. Students were instructed to raise their hands when they finished the worksheet and the instructor would call on three of these individuals, with different z_1 values, to plot their results on a large graph on the white board at the front of the room.

After the three different sets of results had been plotted on the white board, one set for each z_1 value, the instructor would ask the class to explain what was happening to these z_1 values when they were multiplied by $e^{i\pi/2}$ and replotted on the same complex plane. She planned to guide the students, as needed, during this discussion. When the correct conclusion was drawn, she would explain to the entire class, the reasoning behind it and write the following expression on the board:

$$e^{i\theta} \cdot e^{i\pi/2} = e^{i(\theta+\pi/2)}$$

multiplication by $e^{i\pi/2}$ = rotation by $\pi/2$

Then the instructor would ask the class how there could be rotation by the angle delta and if necessary guide them toward the correct answer: multiplying $e^{i\theta}$ by $e^{i\delta}$ rotates $e^{i\theta}$ by the angle δ. She wanted the class to come to a general conclusion based on the discussion and analysis of their graphs. This general conclusion, which was the new concept she wanted to introduce to the students, would then be written on the board in the following way:

"multiplication by $e^{i\delta}$ = Rotation by δ"

bringing an end to the *Phases I* activity. Immediately following the *Phases I* activity would be the *Phases II* activity.

Actual Implementation and Assessment

Overall the implementation of the *Phases I* activity went according to plan and was well received by the students. The majority of the class agreed the worksheet *Phases I* was helpful in their learning process and looked favorably upon the classroom discussion that followed the worksheet. Because the worksheet was short and required only familiar material, students were rarely stuck and most of them finished in the allotted time. The classroom discussion comparing results from different individuals was vital to the success of the activity as it served to bring out the concept under study.

Of the several different teaching styles used up through day 4 in Preface 1, the majority of students surveyed viewed the combination of lecture, class discussion, and worksheet as one of the most helpful ways to learn concepts. One student commented, "The lecture and worksheets were good in the fact that questions were quick to be answered." Another student remarked, "I think discovery worksheets prior to lecturing (are one of the most helpful instructional methods). It helps me to bang my head against something for awhile, then get either a solution that had evaded me, or an insight that I may have missed in my solution. This seems to stick with me longer." It was also suggested by a student that lectures be made shorter, possibly by breaking them up with worksheets.

The format of the *Phases I* activity allowed students to complete a worksheet containing familiar methods, then struggle as a class to conclude a concept based upon this worksheet. In so doing, the students' curiosity was heightened so that when the class was drawn together to close with a lecture drawing out the main concept under investigation, one might expect students' comprehension and retention to be better than if they had not struggled. Oddly enough, there was no consensus among the students as to whether they preferred a worksheet prior to lecture and discussion, similar to *Phases I* activity, or lecture prior to a worksheet, similar to *The Complex Plane* activity.

Several students did not feel that it was a worthwhile learning experience when volunteer students drew graphs and functions on the board. Unfortunately the reasons for these responses were not given, though the instructor identified these students as being shy and likely to be uncomfortable being put on the spot in front of their peers. It should be noted that the students were not told ahead of time that several of them would be expected to go plot their results on the front board.

One student did seem to contradict herself in her survey. She viewed the entire Phases I activity (to include the "discovery" worksheet) as helpful and a worthwhile learning experience but listed "discovery" worksheets completed prior to lecture as a less helpful instructional strategy for learning the concepts under investigation. She preferred lecture followed by group work. This female claims to be comfortable working in groups and comfortable engaging in classroom discussions, yet according to the instructor, came into the program with a very weak background in math and physics.

The students had mixed feelings about the completion of worksheets as a group versus completing worksheets independently. It is interesting to note that although students were free to consult with their neighbors, most of them completed their worksheets individually. In addition, all of the students mentioned they felt comfortable asking questions of the instructor and/or TA during the Phases I activity, yet several of them expressed negative feelings toward having volunteers draw on the board and participating in class discussions.

It was evident from the instructor-guided class discussion, students' surveys and students' worksheets that the majority of the class had met the instructors objectives for the *Phases I* activity. Although the activity was implemented a day later than scheduled, it was completed within the estimated time frame and went essentially according to plan.

NAME ______________________ $z_1 =$

Preface 1 Phases

1) You will find a value of z_1 handwritten at the top of this worksheet. Plot this value of z_1 in the complex plane, using the graph paper provided.

2) Multiply your value of z_1 by $e^{\frac{i\pi}{2}}$ to find another complex number $z_2 = z_1\, e^{\frac{i\pi}{2}}$.

3) Plot z_2 on the same complex plane as before.

Profile: *Phases II* Activity

Objectives

The instructor had the following objectives for the activity:

- **Primary Objective:** Given a complex number of the form $e^{i\theta}$, the students should be able to recognize that multiplication by $e^{i\pi/6}$ shifts the argument of the real (and imaginary) parts of the complex expression to the left by $\pi/6$ on the horizontal θ axis.

- **Closure Objective:** From class discussion and graphical work on the front board, the students will be able to recognize and distinguish between the two different graphical interpretations of phase shift described below:

    - (To be achieved in the previously implemented *Phases I* activity) The first interpretation investigates how multiplication of $e^{i\theta}$ by $e^{i\delta}$ gives $e^{i\theta}e^{i\delta} = e^{i(\theta+\delta)}$ and causes a phase shift (or rotation) of the complex number $e^{i\theta}$ by the angle δ in the complex plane.

    - (To be achieved in the *Phases II* activity) The second interpretation investigates how $\text{Re}(e^{i\theta}) = \cos\theta$ changes when $e^{i\theta}$ is multiplied by $e^{i\delta}$ to give the following result:

      $\text{Re}[e^{i\theta} \cdot e^{i\delta}] = \text{Re}[e^{i(\theta+\delta)}] = \text{Re}[\cos(\theta+\delta) + i\sin(\theta+\delta)] = \cos(\theta+\delta)$, which is a $\cos\theta$ curve shifted to the left by the amount δ, for $\delta > 0$, along the horizontal θ-axis on a rectangular coordinate system with real axes.

Planned Implementation Strategies

This activity would follow immediately after the *Phases I* activity and build upon the *Phases I* activity. It would consist of a student designee plotting on the front board; then a short (approximately five minute) classroom discussion of the plot(s), guided and facilitated by the instructor, but requiring student involvement. The activity was transitional in nature because it began with techniques familiar to the students then progressed to investigate new material.

The groundwork for the *Phases II* activity would be laid earlier in the day through the *Phases I* activity and a brief lecture given by the instructor before the *Phases I* and *Phases II* activities. This lecture would remind students that Re($e^{i\theta}$) is $\cos\theta$, information needed for the *Phases II* activity. Earlier completion of the *Phases I* activity would be necessary in order to achieve the second objective. In addition, it would be assumed that the students understood how changing the argument of a trigonometric function changes the graph of that function. (basic College Algebra/Trigonometry reviewed in a homework assignment from earlier in the week)

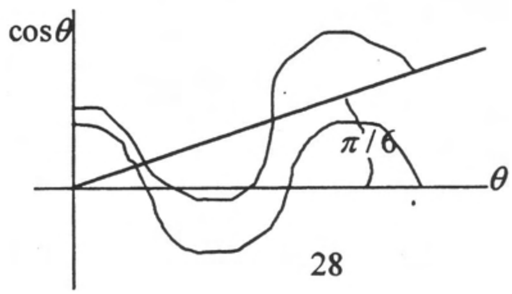
The instructor planned to call student volunteers to the front board to plot Re($e^{i\theta}$) then Re($e^{i\theta}e^{i\delta}$) on coordinate axes with θ on the horizontal axis. Then, through questioning, she would guide the class in a comparison of the two graphs [Re($e^{i\theta}$) and Re($e^{i\theta}e^{i\delta}$)] until they arrived at the correct conclusion. Immediately following the conclusion she would refer to the concept investigated in the *Phases I* activity, that "rotation by δ = multiplication by $e^{i\delta}$", and discuss the differences between the two representations of Phase Transformation, bringing closure to the *Phases II* activity.

Actual Implementation and Assessment

Overall the *Phases II* activity went essentially according to the instructor's plan with the exception of a valuable learning experience brought about by one student's misconception. The instructor accurately estimated the time for completion of the activity although the original plan was for the activity to be implemented a day earlier. The students' direct involvement in this activity opened the door to errors that might retard the class progress. However, since the students were not free to work individually but were being guided by the instructor, their time spent going down the wrong path could be closely controlled.

When asked to draw Re($e^{i\theta}$) on the axes on the front board, the first student volunteer began by writing Euler's formula ($e^{i\theta} = \cos\theta + i\sin\theta$) on the board, then proceeded to draw the correct curve, $\cos\theta$, on the board. The significance of the students' intermediate step of writing down Euler's equation should not be minimized, as it sheds light on how this student accessed previously constructed knowledge, and was a necessary part of the thought process.

The activity began deviating from the instructor's plan when the second student volunteer was asked to plot Re($e^{i\theta}e^{i\pi/6}$), but it resulted in a valuable learning experience for everyone. The student referenced an expression the instructor had just written on the board, $e^{i\theta}e^{i\pi/6} = e^{i(\theta+\pi/6)} = \cos(\theta+\pi/6) + i\sin(\theta+\pi/6)$, and drew the cosine curve rotated by $\pi/6$ radians, in the following way:



The student had wrongly assumed that when $e^{i\theta}$ is multiplied by $e^{i\pi/6}$, the real part of the resulting expression, $e^{i(\theta+\pi/6)}$, plotted verses θ would behave graphically just as the complex expression did plotted with θ as a parameter. The student failed to see the distinction between the different roles of θ in the two, thus producing a counterclockwise rotation of the entire cosine curve by the angle $\pi/6$. Through instructor questioning and explanation, this student apparently realized the error in their thinking, but allowed a third volunteer to correct the plot on the board.

The third volunteer correctly realized that the resulting expression was just the cosine curve shifted horizontally and proceeded to draw a cosine curve shifted to the right by $\pi/6$. Evidently he did not understand that graphically, $\cos(\theta + \pi/6)$ is the cosine graph shifted to the left by $\pi/6$, another instance when the students did not have the simple trigonometry/college algebra background knowledge expected of them. The correct graph was finally drawn and through class discussion and comparison of the graphs on the board, the instructor explained the difference between the two types of phase shift they had encountered in the *Phases I* and *Phases II* activities.

The majority of students felt the classroom discussion, involving student volunteers drawing graphs and functions on the board, deepened their understanding of the concept of phases and was a worthwhile learning experience. All of the students surveyed agreed they felt comfortable asking questions of the instructor and TA during the activity. Given this consensus, it is interesting to note that when asked which methods (such as class discussion, group work, individual work, Maple worksheets, etc) were most useful in learning the material, none of the students mentioned class discussions. Instead several of them mentioned that having volunteers draw on the board and class discussions were the least helpful methods in their learning of the material. These students were identified by the instructor as being shy and likely to be uncomfortable being put on the spot in front of their peers.

Although there was not a written assessment of whether the students met the instructor's objectives, the entire class heard the discussion between the instructor and the student volunteers at the board and the instructors spoken comparison of the two representations for Phase Transformations. The degree to which students comprehended the discussion is unclear. Based upon the above mentioned statements and other purely anecdotal feedback from the observer, instructor, TA, and students, it seemed evident that most of the class understood the distinction between the two different types of phase transformation under investigation. In a purely qualitative sense it was apparent that the instructor's objectives had been met for the majority of the class.

The Study of Power Series

A significant portion of Preface 1 involved the study of power series. The following activities were developed to investigate this topic on Day 4 and Day 5 of Preface 1: *Power Series Coefficients* and *Convergence of Power Series*. The instructor had the following instructional goals for these two days:

Goal 1: The students would come to understand that where a series converges is not necessarily where the truncated series is a good approximation.

Goal 2: The students will realize a power series expansion is a representation of a function in the region in which the function converges. But outside of that region the power series is nonsense. The function, however, may be well defined.

Goal 3: The students will begin thinking at a higher mathematical level.

Goal 4: The students will gain experience with Maple.

Profile: *Power Series Coefficients* Activity

Objectives

- **Review Objective A:** This activity will aid the students in a review of the language of power series including terms such as "coefficient", "expanding about a point", and "correct to fourth order".
- **Review Objective B:** The brief lecture prior to the *Power Series Coefficients* worksheet will aid the students in a review of how to find Taylor Series coefficients using derivatives.
- **Primary Objective:** Presented with a derivation of how to calculate the coefficients of a power series, the students will be able to find the coefficients for the power series expansion of $\cos(\theta)$, which are needed for the next activity involving the Maple worksheet *Power.mws*.
- **Secondary Objective:** Given a function, the students will be able to expand that function about a point other than zero.

Planned Implementation Strategies

The *Power Series Coefficients* activity was planned to begin and end on Day 4 of Preface 1 (1 hour and 50 minute class). The activity would consist of a brief lecture by the instructor, calculating the coefficients for a power series, then students' completion of the *Power Series Coefficients* worksheet, followed by a brief lecture by the instructor wrapping up the first half of the worksheet (expanding about $\theta_\circ = 0$). The self-explanatory worksheet, developed by the instructor, was estimated to take about 15 minutes for completion.

It was expected that most students had seen the power series expansions in their calculus course, therefore the groundwork for the activity was laid in students' prerequisite coursework. During a brief lecture at the beginning of the activity the instructor would do a general review of power series, deriving how to calculate the coefficients of a power series. The students would then complete the *Power Series Coefficients* worksheet, requiring them to utilize the techniques just derived in a guided practice session.

For the worksheet portion of the activity students would be counted off by threes and form groups around the classroom. Each student would be given a *Power Series Coefficients* worksheet and asked to complete it individually, seeking help from group members when needed. During the worksheet completion, the instructor and TA would be walking around observing the students, answering and asking them questions. They would specifically check their coefficients and whether they had the series expansion written to the correct order.

Students would be asked to raise their hands when they were finished with the worksheet, then once the majority of the class had completed the worksheet, the instructor would bring the entire class together for a wrap-up. The wrap-up of the activity would consist of the instructor explaining what the coefficients for the power series expansion of $\cos(\theta)$ should be when expanded about the origin. These coefficients would then be used in a Maple worksheet called *Power.mws* that would plot various approximations of the series together with the $\cos(\theta)$ curve.

Actual Implementation and Assessment

Most of the activity went according to plan, although the instructor and TA were very surprised at the difficulty students experienced. When developing the activity the instructor had not anticipated students struggling with the *Power Series Coefficients* worksheet because the material contained on it should have been review and her brief lecture provided them with the tools needed to complete it. Based upon the student-teacher interaction during the activity and analysis of the students' worksheets, the researcher concluded that the notation and language used in the worksheet significantly contributed to students' confusion. They struggled in applying the derived formula for the coefficients, in terms of the variable z, to the function under expansion given in terms of θ and they confused the order of the series approximation with the number of terms in the series.

The unanticipated student problems caused the completion of the worksheet to take longer than expected. The lack of time resulted in some students finishing the worksheet for homework. Several students commented that because it had been so long since they had seen the subject matter, they needed more time for the activity. The majority of the class successfully finished the first half of the worksheet, dealing with expansion about $\theta_{\circ} = 0$, yet several students did not have time to finish the second half of the worksheet, expansion about $\theta_{\circ} = \pi / 6$. The instructor wrap-up explained the solution to the first half of the worksheet, leaving the students without immediate feedback on their expansion about $\theta_{\circ} = \pi / 6$, for which they had not seen an example. It should be noted that about half of the students' worksheets turned in the following day contained incorrect expansions about $\theta_{\circ} = \pi / 6$. These students had the correct coefficients but did not write the series as powers of $z - \pi / 6$. Their mistake could have been due to a number of factors but was most likely due to students being unfamiliar with expansions about a point other than the origin. It seems that they understood how to calculate the coefficients using derivatives since they arrived at the correct coefficients for both the expansion about $\theta_{\circ} = 0$ and $\theta_{\circ} = \pi / 6$. Since these student errors went unnoticed and uncorrected during the activity this indicates a weakness in the way the activity was structured.

As the instructor had expected, the majority of the students viewed the *Power Series Coefficients* activity as a review, although about half of the class said they needed further instruction on the method for calculating power series prior to doing the worksheet. This might indicate that the opening lecture on power series, preparing students to complete the worksheet, could have been more extensive, possibly including more examples worked out in greater detail. After the activity, the instructor was unsure whether the activity was appropriate for an in-class, group activity, suggesting that there might be a better way to spend class time. The first part of the worksheet, could have been assigned for homework the night before because it involved the expansion of $\cos\theta$ about the origin, which was fairly straightforward. This would allow more class time to focus on the portions of the worksheet where students were error prone. In a discussion following the activity, the instructor concluded with the comment, "small group work for review is not good use of time." A year later the instructor chose to again include this activity in Preface 1 because she saw it as a necessary review. Implementing it the second time, the instructor changed her mind. She planned to have students complete only the expansion about $\theta_\circ = 0$ during class, leaving the expansion about $\theta_\circ = \pi / 6$ for homework.

The students had mixed feelings about the effectiveness of the structure of the *Power Series Coefficients* activity, i.e. lecture first, then worksheet following. Just over half of the class viewed the approach of lecture first then worksheet following as effective in building their understanding of power series, while the other half felt this teaching style was somewhat effective for their learning.

In reflecting on the entire week of Preface 1, the students were asked to list what teaching technique, of the many used in Preface 1, was a most helpful technique for their learning the concepts and methods under investigation. Several students in the class answered this question mentioning lecturing prior to a worksheet/exercise, as in the *Power Series Coefficients* activity. It is significant that these same students shared the following: a weak background in math/physics and generally quiet in the classroom. In addition, most of them were not fond of teaching methods involving volunteers drawing on the board.

Overall the activity did not go as smoothly as planned, causing the objectives not to be met in the planned way. About half of the worksheets turned in by the students the day after the activity showed they were still not familiar with the language of power series, indicating that review objective A had not been met by all students. Students' successful calculation of the Taylor (power) series coefficients in the first half of the worksheet (expanded about $\theta_\circ = 0$) using derivatives showed evidence that review objective B had been achieved by essentially the entire class.

The TA and instructor agreed that the majority of students successfully completed the first half of the worksheet in their groups, prior to the instructor's wrap-up. They proceeded to use their results from the worksheet in the Maple worksheet *Power.mws* thus meeting the primary objective for the activity. Based upon students' worksheets turned in the following day, only about half of the students met the secondary objective.

Preface 1 Power Series Coefficients

Consider the power series

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$$

expanded around the point z_0. In lecture we just derived that the coefficients are given by

$$a_n = \frac{1}{n!} f^{(n)}(z_0)$$

1) Find the first five nonzero coefficients for $\cos(\theta)$ expanded around the origin.

2) Write out a series approximation, correct to fourth order, for $\cos(\theta)$ expanded around the origin.

$\cos(\theta) =$ ______________________________

3) Find the first four nonzero coefficients for $\cos(\theta)$ expanded around $\theta_0 = \frac{\pi}{6}$.

4) Write out a series approximation, correct to fourth order, for $\cos(\theta)$ expanded around $\theta_0 = \frac{\pi}{6}$.

$\cos(\theta) =$ ______________________________

Profile: *Convergence of Power Series*

Objectives

The instructor's objectives for the activity were as follows:

- **Review Objective A:** Through completing the worksheet, the students will gain practice with complex number algebra, and complex number generalizations of absolute values.
- **Review Objective B:** On completion of the worksheet, the students will see an example of the equation for a circle in complex notation, $|z-a|=r$ and recognize the center, a, and radius, r.
- **Primary Objective:** Using the ratio test, which gives the region of convergence for a series, the students will understand why the region is always a circle, centered about the expansion point, that extends out to the nearest singularity.

Implementation Strategies:

The *Convergence of Power Series* activity was planned to begin and end on Day 5 (50 minute class) of Preface 1. The activity would consist of a brief lecture given by the instructor followed by students' completion of the *Convergence of Power Series* worksheet, then an instructor wrap-up of the main concepts in the worksheet. The self-explanatory worksheet, developed by the course instructor, was expected to take about 30 minutes for completion.

Some of the groundwork for the activity would be laid earlier in the week when students learned how to manipulate complex numbers algebraically and through homework assignments dealing with the equation of a circle and the absolute value of a product. Students should have prior knowledge of series and convergence for real numbers from their prerequisite courses. The instructor would give a brief lecture reviewing how to determine where real series converge using the ratio test, then generalizing this to complex numbers. She would then extend the convergence test to series with complex numbers and write this result on the board. The students would use her result for the special example of power series in the next portion of the activity, completing the Convergence of Power Series worksheet.

The students would be grouped randomly in fours and given their own copy of the *Convergence of Power Series* worksheet. A worksheet would be completed individually by each member of the group and discussion between group members would be encouraged. The instructor also made it clear that she or the TA would be available to answer questions the group could not answer. While students completed the worksheet, the instructor and teaching assistant (TA) would be circulating the room, asking questions which checked for student understanding and answering students' questions.

Following the worksheet the instructor would bring closure to the activity by summarizing the results students should have found in the worksheet. She would expand on the geometrical meaning of the values of z for which the power series converges and give several examples. The instructor would also discuss some new information on the fact that power series converge for values of the argument that are inside a circle out to the nearest singularity and centered about the expansion point.

There are several reasons why the instructor was motivated to incorporating these concepts into an in-class, group worksheet type of activity. Realizing that the convergence of a power series is a difficult concept she thought that lecturing over the topic, as she has in past years, would have spent too little time on it and resulted in many students trivializing the ideas and methods. She felt that by having students do the complex algebra themselves, the derivation would be slowed down, allowing students to focus more on the geometrical meaning of convergence of a power series. From observing students in the past she has noticed that students working together seem to see the big picture better than those working alone, thus she decided to implement the activity as a group exercise instead of an individual exercise.

In addition, it should be noted that recognizing that the students would probably struggle in completing the worksheet, the instructor believes students might benefit from a brief explanation of the motivation behind the activity design. For example she might say, "This is a challenging concept and may take a few days to sink in." By students knowing what to expect in terms of difficulty level for a particular activity, it is likely that they will not be as easily discouraged when they are struggling.

Actual Implementation and Assessment

Overall the *Convergence of Power Series* activity went according to the instructor's plan. However, the difficulty level of the worksheet portion of the activity seemed to cause the students to spend a significant portion of time struggling. As a result none of the students were able to finish the worksheet in the allotted time.

The instructor had not planned to pull the entire class together until they were near the end of the worksheet, but her plan quickly changed when she realized the students were struggling with how to begin the worksheet. While working on question 1 of the worksheet, the groups tried many different strategies yet the majority of them had trouble determining how to apply the ratio test to the general power series of *f(z),* which they had seen on their homework the day before. The instructor had expected students' to have such a misconception but it was more widespread than anticipated. Recognizing their struggle the instructor eventually pulled the entire class together and briefly lectured them on this point, preparing them to move on to the next step. The observer said that it was apparent that most students' confusion was helpful in their learning and really got their wheels turning. He commented that the instructor's lecture seemed to clear up the majority of students' confusion.

Before the instructor interceded, the majority of students were actively participating in their group's discussion, but after the instructor pulled the class together this seemed to change. In several groups, one or two individuals began to take the role of leading the discussion. This responsibility was shared more equally in the remaining groups, whose members checked their results with each other along the way.

When the instructor finally pulled the class together for a wrap-up of the worksheet, most of the groups were headed in the right direction on the worksheet but none of them had finished it. The majority of the groups had completed most of question one, but they had not recognized that the power series converged inside a circle, the main concept under investigation. Most of the students had worked through number one and arrived at the following equation, when the instructor pulled them together for the wrap-up:

$$\lim_{n\to\infty}\left|\frac{C_{n+1}(z-a)}{C_n}\right|<1$$

Although no students had finished the worksheet, the instructor and observer agreed that essentially all of the class had struggled constructively and were far enough on the worksheet that the instructor's wrap-up was still effective for their learning. When the instructor wrapped up, she solved the problem and tied the concepts of power series approximations together with the convergence of a power series, pointing out that where the truncated series converges is not necessarily where the complete series converges. The students' struggle with the worksheet supported the instructor's idea that lecturing on the material allows students to trivialize it.

All of the students agreed their group struggled a lot as they worked through the worksheet, seeking to recognize that a power series converges inside a circle, yet the students had mixed feelings as to whether their struggle was constructive. About half of the class viewed their group's discussion as productive in their learning from the worksheet, while the other half did not. It is interesting that most of these students also claimed their group discovered the radius of convergence of a power series before the instructor's wrap-up, which does not agree with the observer's report of how far students got on the worksheet. If the students' claim is correct then this might indicate these students don't consider struggling to be productive unless they eventually arrive at the desired results. The instructor, however, believes the students' claim that they finished the worksheet might have been because they felt a sense of completion and believed they understood what it was they were expected to learn. It should be noted that based upon student's surveys, there was not an obvious connection between those students who viewed their group's discussion as unproductive and these same students' comfort with group work.

The following student comment might shed some light on why students struggled with the worksheet. He said, "the worksheet was okay but the main difficulty was that none of us could recall our Math 253. We weren't just learning complex series, we were relearning series in general." Although the students struggled with the *Convergence of Power Series* activity, it is significant that several students expressed enthusiasm about the activity and viewed it as the most powerful math/physics realizations they had seen during Preface 1.

Reflecting on the activity, the course TA had the following thoughts. He was not sure how effective it was to use class time to cover the ratio test since it should have been review for most of the class. He also felt the worksheet might be more effective if it were structured differently. Since questions 1 and 2 were related and question 3 stood alone, he suggested that question 2 be made into part b of question 1. He mentioned that the open-ended style of question 2 left many students confused about what the instructor was really looking for. Finally, he believes that students significantly deepen their understanding of a concept when they learn a method and immediately apply it to a specific example. Therefore the TA saw the activity as being compromised because no students were able to complete question 3, which entailed finding the circle of convergence for a specific example.

It was unclear whether the *Convergence of Power Series* activity met all of the objectives. Overall the instructor felt the objectives had been met, based upon her interaction with the students during the activity. All of the students practiced their complex number algebra and generalizations of absolute values when they worked through the first steps of question 1 on the worksheet, providing evidence that review objective A was met by all of the students. There was not a direct assessment of whether review objective B and the Primary objective were met by every student in the class, although the students were exposed to the concepts involved in both objectives through the instructor's wrap-up lecture.

Preface 1 Convergence of Power Series

The ratio test is a standard test for the convergence of a series. Recall the ratio test says to divide the $(n+1)^{th}$ term of the series by the n^{th} term. Then take the limit of the absolute value of this ratio as n goes to infinity to obtain the quantity ρ as shown below:

$$\rho = \lim_{n\to\infty} \left| \frac{c_{n+1}}{c_n} \right|$$

If

$$\rho \begin{cases} < 1 & \text{the series converges.} \\ = 1 & \text{the test is indecisive.} \\ > 1 & \text{the series diverges.} \end{cases}$$

Consider the power series for an arbitrary function $f(z)$ expanded around the point a.

1) Use the ratio test to find a condition on $|z-a|$ for which the series is convergent. (Hint: remember $|w_1 \, w_2| = |w_1||w_2|$ for any complex expressions w_1 and w_2)

2) Describe (in words) the values of z for which the power series converges?

3) Find the values of z for which the power series (around the origin) for $\cos z$ converge?

The Study of Linear Transformations

Note: This activity was implemented in Preface 2, which was one term after Preface 1.

A portion of Preface 2 involved the study of matrix manipulations and the use of matrices to perform linear transformations on vectors. The *Linear Transformations* activity was developed to introduce students to these topics and provide several examples of linear transformations. The instructor had several goals for the week of Preface 2 and the goals related to the *Linear Transformations* activity are listed below.

Goal 1: To use the concrete example of rotations to introduce the concept of states and transformations on states.

Goal 2: To review matrix manipulations from linear algebra and to learn common physics symbols, such as bra-ket notation, for these operations.

Profile: *Linear Transformations* Activity

Objectives

The instructor and TA had the following objectives for the activity:

- **Primary Objective:** Given specific examples of simple linear transformations, the student will be able to categorize what these transformations do to vectors, such as dilate, or rotate.
- **Secondary Objective:** Given various matrices, students will review and practice matrix multiplication.

Planned Implementation Strategies:

The *Linear Transformations* activity would begin and end on Day 2 of Preface 2 (1 hour and 50 minute class). The activity would consist of a brief lecture introducing the concepts under investigation, students completion of the *Linear Transformations* worksheet, students graphing on the front board, followed by a guided class discussion highlighting the concepts under investigation (such as rotations, inversions, etc.). The course teaching assistant (TA), who developed the activity, would teach it under the instructor's supervision. They expected the activity would take about 20 minutes for completion. The activity was transitional in nature because it would begin with techniques familiar to the students then progress to investigate new material.

The groundwork for the *Linear Transformations* activity was laid on Day 1 and through a handout titled "Linear Algebra by Example," which the students were to complete prior to the activity. The handout reviewed and introduced various matrix manipulations and was discussed briefly in class on Day 1. Finally, viewing a vector as a column matrix was introduced and set the stage for the ideas to be studied during the week of Preface 2.

The methods required for the completion of the *Linear Transformations* worksheet had been seen previously by the students. These methods were matrix multiplication and the ability to view column matrices as vectors and graph them. The activity would be introduced through a brief lecture describing the context of the activity and include the following points:

- An introduction to the idea that vectors are connected to both linear algebra and physics.
- The posing of this question to be investigated in the activity: If column matrices are vectors then what role do square matrices play in physics? (this aids students by helping them know what to look for as they complete activity)

The structure of the class during the activity would be as follows. The students would be randomly grouped in threes or fours with each student receiving a *Linear Transformations* worksheet, a piece of graph paper, and colored pencils (red, green, blue, yellow, purple) corresponding to the five different vectors in the worksheet. Each group would be assigned a different matrix, A_i, from the worksheet to be used as an operator on each of the five vectors. In addition the TA would graph the original vectors on the front board and draw coordinate axes for each group, labeling each with the corresponding matrix operator A_i. As the groups are working on the *Linear Transformation* worksheet, the TA and instructor would be walking around answering questions and overhearing group discussions, adding comments when appropriate.

After completing the self-explanatory worksheet, including individuals graphing the initial and transformed vectors on their graph paper and noting any differences between the initial and transformed vectors, one member of every group would be asked to draw their results on his/her group's coordinate axes at the front board. A guided classroom discussion and comparison of results facilitated by the TA would then wrap up the activity.

In discussing the results on the board, the TA would take suggestions from each group about what their transformation was, while the rest of the class and the TA commented on their response, making any corrections. The students would be guided toward looking in detail at how the initial vectors were affected by multiplication by each group's matrix. An integral part of the activity would be the class comparison of the results of various groups leading into a discussion of length changes and any correlation between the transformation and the determinant of the matrix A_i. During the wrap-up the TA would bring out the physical relevance of the sign of the determinant of A_i and the class would participate in recognizing patterns between this sign and the type of transformation. Finally, the question posed at the beginning of the activity would be answered through a discussion highlighting the following concepts: (a) an operator is a square matrix, (b) operation on a vector results in a transformed vector.

Following this activity would be more extensive lecture and discussion of rotation matrices. The activity would serve to introduce the concept of rotations through a display of examples that would hopefully intrigue and motivate the students for the lecture material, which would follow. Many of the newly introduced techniques would be practiced on the homework that evening.

Actual Implementation and Assessment

Overall the *Linear Transformations* activity went according to plan. The instructor, TA, and researcher/observer all felt the activity was successful and went according to plan. One of the most valuable aspects was that each student was actively involved during the worksheet portion of the activity and each group was represented when the results were discussed. The individuals in each group completed the worksheet individually, consulting with their fellow group members when they had questions.

Another benefit was that the activity gave each group practice using the same techniques with their different matrices, A_i, providing the class with the opportunity to learn about the results from each group. In other words, the seven groups produced seven different transformation examples that yield seven different graphs each of which could be compared with the graphed initial vectors to arrive at a conclusion. The above described technique was very effective use of class time since each group's resulting transformation was used as an example for the entire class to see different types of Linear Transformations.

It was obvious to the observer that groups needed instruction on how they were to work together. Once the students were randomly placed in their groups they neglected to form circles and during the first five minutes of completing the worksheet they did not talk with their fellow group members. It is possible that since the first part of the worksheet was over familiar material, they did not need to seek help or discuss with other members of their group, but the more likely reason was that they were not encouraged by the instructor to talk among themselves. When the instructor walked about five minutes into the worksheet and encouraged the class to interact with their group and form circles, they quickly began working together.

It should be noted that, based on observation, students seemed more comfortable working in groups and engaging in the classroom discussion during this Preface 2 activity than during many of the Preface 1 activities requiring group interaction and classroom discussions. The situation described above could be due to a number of factors but based upon student surveys may be directly related to students not knowing one another very well during Preface 1.

The entire Linear Transformations activity took 20 minutes, about as long as expected. One student who had completed the traditional junior year curriculum made a noteworthy comment in an interview one week after completing Preface 2. When asked what she did and did not like and where she learned the most in Preface 2, she said most of Preface 2 was review but mentioned she liked the Linear Transformation "lab" stating that it was really simple and good. She continued by saying how valuable *the Linear Transformations* activity was because it introduced a set of examples early in the week, enabling the professor to refer back to them for the entire week.

The wrap-up of the activity made it evident that the primary objective had been met because when representatives of each group plotted their group's results on the board the majority of the groups correctly identified their transformation as a rotation, a reflection, a dilation, etc. One group had to reason with the class as to why their transformation was not a rotation by 180 degrees but instead was a reflection across an axis. In addition, the class comparison of the results of each group indicated that learning had taken place. Later in the week it became evident that the secondary objective had been met since subsequent homework assignments had students practice matrix multiplication and required they have an understanding of the Linear Transformation activity and what rotation matrices do. Further confirmation that the primary objective had been met came when the instructor referred back to the *Linear Transformations* activity during the remainder of the week, and students' responses showed they were familiar enough with these concrete examples that they could recognize them.

It is important to note that the activity did seem to grab the students' attention and interest them in the lecture that followed, which involved a thorough investigation of rotations.

Linear Transformations

by Jason Janesky & Corinne Manogue

1) Using colored pencils, draw the initial vectors below on the top half of the graph paper provided.

$$|\text{red}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |\text{green}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad |\text{blue}\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad |\text{yellow}\rangle = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad |\text{purple}\rangle = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

2) Each group will be assigned one of the following matrices. Operate on the initial vectors with your group's matrix and graph the transformed vectors on the bottom half of the graph paper provided.

$$A_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad A_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad A_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad A_5 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A_6 = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \quad A_7 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad A_8 = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad A_9 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

3) Make note of any differences between the initial and transformed vectors. Specifically, look for rotations, inversions, length changes, anything that is different. Your group should be prepared to report to the class about your transformation.

Hypotheses

Hypotheses related to in-class activities:

1. Valuable/effective in-class activities can be developed which require students to compare and analyze graphs to recognize a concept under investigation.
2. In-class activities that require students to apply the same technique to different examples are effective in deepening students' understanding of concepts through the comparison of many examples.
3. Use of a transitional, warm-up activity engages the students in thinking needed for understanding new concepts.

Hypothesis related to group activities:

4. Groups of three students are effective for getting the vast majority of students involved in an activity.

Hypotheses related to activities containing worksheets:

5. When worksheets are used to guide student development, periodic lecture/discussion by the instructor is valuable for summarizing the concepts being developed and resolving student misconceptions.
6. Worksheets are more effective when they are short, containing several (approximately three) questions which all relate to one central concept as compared to longer worksheets covering multiple concepts.
7. Worksheets are less effective when they require students to utilize knowledge from prerequisite courses.

Discussion of Hypotheses

In triangulating the profiles for the six activities, the researcher found it helpful to do a comparative ranking of her believed success of the activities. The ranking is based upon the researcher's overall impressions of the activity using the data collected and assembled in the profile for each activity. The activities were given the following rankings (in increasing order) on the lichert scale seen below.

1	2	3	4	5
less successful		moderately successful		more successful

Power Series Coefficients Activity = 2

The Complex Plane Activity = 2

Phases II Activity = 3

Convergence of Power Series Activity = 4

Phases I Activity = 5

Linear Transformations Activity = 5

Triangulation of the profiles requires that evidence supporting a hypothesis be found in more than one profile. Thus hypotheses were drawn when commonalties and differences were seen in several activities.

The *Phases I*, *Phases II* and *Linear Transformations* activities were considered moderately to more successful. Common to all of the above mentioned activities is they involved students plotting, comparing and analyzing graphs as they were guided toward recognizing a new concept. The first hypothesis was drawn from the recognition of the above trend.

The second hypothesis was drawn from the success of the *Phases I* and *Linear Transformations* activities. Both activities involved students in recognizing a concept. In the *Phases I* activity students investigated phase rotations while in the Linear Transformations activity students learned that when a square matrix operates on a vector, the vector is transformed. During each activity student groups performed calculations for different examples and presented their results to the class for comparison. The researcher believes the format of the *Linear Transformations* activity to be especially valuable because it allowed the class to see a set of examples that were referenced when the material was studied further.

Hypothesis three was drawn from a pattern seen in *The Complex Plane*, *Phases I* and *Linear Transformations* activities. All of these activities began with students using familiar graphical or calculation techniques, which transitioned students into the study of new material.

Common to *The Complex Plane*, *Phases I*, *Power Series Coefficients*, and *Linear Transformations* activities is that they all involved students working cooperatively in groups of three. From this pattern the fourth hypothesis was drawn. It is significant that when placed in groups of four during the *Convergence of Power Series* activity the interaction of several student groups might indicate that groups of three are more effective than groups of four.

Students' learning during all activities involving worksheets was repeatedly enhanced by the instructor's periodic intervention to get groups back on track or to summarize and draw out the main points of an activity. It was from this pattern that hypothesis five was formulated. Because students need adequate time to complete portions of the worksheet and struggle, the instructor carefully gauged when to intervene by having herself and the teaching assistant circulate the room, monitoring students' progress.

The Complex Plane worksheet was the longest worksheet implemented and was less successful. It covered three concepts through a series of 13 steps that involved calculations, graphing and conceptual/comparison questions. More successful activities such as the *Phases I* and *Linear Transformations* activities were comparatively shorter and covered just one concept through a series of two to three steps usually involving one calculation, one graph and one conceptual/comparison question. Hypothesis six was drawn from the pattern noted above.

Finally, the seventh hypothesis was drawn based upon a commonality seen between two of the less successful activities. In completing *The Complex Plane*, and *Power Series Coefficients* worksheets, students had difficulty recalling and using mathematical techniques learned in prerequisite courses.

Conclusion

This qualitative research project sought answers to the following question: When, where and how is it effective to integrate interactive activities into lecture in upper division physics theory courses? The six activities investigated were implemented in either Preface 1 or Preface 2 of the Paradigms in Physics curriculum reform presently underway at Oregon State University. The researcher collected data from a variety of sources and assembled it into a profile for each activity. Using the method of triangulation the data were analyzed and six hypotheses for further study were drawn.

As the researcher compiled data, wrote profiles and developed hypotheses, she became aware of several ways the project could have been improved to possibly yield more and better hypotheses. The following paragraph contains changes that the researcher would make if the study were conducted again. The goals and objectives for an activity would be explicitly laid out prior to designing the activity. The same observer would be used for all activities with the observer noting and recording the same aspects of each activity. Use of a video camera to record students' interaction for later comparison with other activities would be valuable. Whenever possible, student feedback would be received through interviews that include the researcher asking a minimum set of pre-determined questions. If student surveys are used, they will be used for each activity and structured similarly. After every activity, the researcher will obtain feedback from the instructor and teaching assistant through a structured interview. Finally, studying sets of activities with similar purposes and designs would provide more evidence for triangulation and therefore more hypotheses.

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Appendix A

General Observations Pertinent to all Profiles

There was no consensus with the students about the preferred order of an activity, such as lecturing prior to application of lecture material in a worksheet or a worksheet where they struggle to realize a concept that will be followed up on in lecture. Student feedback revealed that several students with a weak background in math and/or physics were not fond of struggling to realize concepts through a worksheet before lecture over these concepts, but instead preferred lecturing prior to application of lecture material in a worksheet. In addition, several quiet and shy students viewed both student volunteers drawing on the board and classroom discussions as ineffective for their learning of the material.

It would be worth investigating the reasons behind students' preferences for the order of an activity, in order to determine if particular types of students learn better when material is taught in a certain order or using specific techniques. One might ask if there are certain groups of students who are not learning effectively from various styles of activities.

Below are two very different perspectives from female Paradigm students who participated in the group activities, which took place during Preface 2. The first female also participated in Preface 1, Paradigm 1, Paradigm 2, and Paradigm 3 activities.

A female student in an interview following Preface 2 reported the following:

She sensed competitiveness within groups and between groups. She suggested that two students be in a group since she liked being in pairs for the Maple worksheets and did not find the Maple activities competitive. She finds that when two males are in a group with her, they are in somewhat of a race with each other and do not take time to discuss the activity, which she sees as one of the points of the group work. She stated the overall group definitely seems to be competitive but is not sure if this is typical or not. She claims that one contribution to competitiveness might be when the professor wraps-up the activity by saying "Does anybody have this…" because everybody wants to be the group to say, "Oh, we do, we're the smart ones."

She does not like the time constraint as she feels the pressure for time in conjunction with competitiveness results in her not being honest with her group about not understanding something for fear of slowing them down. Often group members do not want to take time to explain things to someone who does not understand because this will slow them down. In addition she claims to notice that if someone is behind they are reluctant to ask another group member for help or explanation for fear of looking stupid. She feels that her willingness to explain things to someone who does not understand has put her at a disadvantage because the third group member usually continues on and then she must catch up by asking the third group member to explain things to her. She claims the third group member usually does not want to explain things because he/she wants to rush on.

Another female student in an interview following Preface 2 reported the following:

She was very comfortable working in groups and sees it as very effective. She prefers groups of three over groups of two since then there is a third opinion. She feels the majority of the students in the class are mature enough for group work and does not think that competition is an issue. She feels everyone contributes quite equally. She is not bothered by the time constraints imposed on the group during an activity.

Procedural Advice

The following recommendations are in response to procedural errors common to various activities but were not appropriate to include in the profiles.

- Worksheets and similar materials should be handed out to the class after giving complete initial instructions orally. Otherwise students tend to get distracted reading the paper in front of them and do not hear the oral instructions.

- In order to reduce competition between students, the majority of the class needs to be done with the worksheet before volunteers from each group are allowed to report their results to the class.

- Having students review concepts and/or methods on their homework before they apply these concepts and/or methods during an in-class activity is a good way to increase their involvement in discussion and reduce time spent off task.

- Group activities involving small groups of students discussing a worksheet that guides them in realizing a concept are an effective way to introduce concepts that will be expanded upon later in a lecture presentation.

- The dynamics and participation of students in groups are improved for each of the following situations:
  - when groups receive instruction and encouragement on how they are to interact with each other.
  - after several group activities have been implemented.
  - when roles are assigned to each student in the group.

- Worksheets that require students to think at a high level might be more effective when students are made aware that the worksheet is expected to be a challenge.

- Worksheets need to be tested prior to student use in order to estimate the time needed for completion.

- The objectives of the worksheet need to be explicitly identified, in advance of development, in order to direct the form of the worksheet.

- The assumptions built into the worksheet design need to match the characteristics of the class.

- Engaging students in discussion by having them present ideas to the class is an effective way to help them clarify their understanding.