Week 3 "Tutorial" – Designing an Electric Field

Goals:

1. Gain confidence with divergence and curl, and understand what they mean physically (L.G. 9)
2. See a relationship between mathematical representation of E-field and graphical (L.G. 2)
3. Understand what charge distribution causes an electric field (L.G. 1)
4. To decide if a mathematically plausible field is physically possible as well. (L.G. 1)
5. Path independence of voltage difference (L.G. 5c)
6. How to choose a "good" voltage reference point (L.G. 5c)
7. See the connection between calculating potential difference and measuring it with a voltmeter. (L.G. 1)
8. Apply physics knowledge to a real-world example.
9. Estimate order of magnitude of a real-world problem (L.G. 5a)

This tutorial is based on:

- "Griffiths by Inquiry" – Section 1, Introduction to Electrostatics
- OSU "Measuring Voltage"

Materials needed: big white boards, dry erase markers, Jacob's ladder demonstration (could watch a video of a Jacob's ladder if demo is not available – youtube.com has several examples)

Tutorial Summary: Students make up their own electric field and write down its vector formulation, sketch it, check that it satisfies Maxwell's equations, and justify the choice of a reference point to measure its potential. Jacob's Ladder is used as a demonstration and students are asked to make sense of the physical behavior they observe.

Written by: Darren Tarshis
Contact: Steve Pollock (steven.pollock@colorado.edu)

A few words about running this tutorial:

The room setup has several small tables, with four chairs each. Each group was given four markers, and a big white board that covered the whole table. This allows students to easily communicate their ideas to each other and to the Learning Assistant (LA). It also helps the LA communicate with the students more effectively.

Students are advised that it should take approximately an hour to complete the tutorial. They are also advised that this is completely voluntary, so they will not be required to turn in the tutorial. However, they are advised that it would be in their interest to fill things out as completely as possible, since many of the topics would also be covered on the homework. The group members are encouraged to work together, and the LA wanders from group to group to see if there are any misconceptions, or if anyone is completely lost/stuck. In the words of Steve Pollock, "We will try our hardest not to give you any of the answers, but we will also try our hardest to make sure you figure everything out for yourself."

After the students work on the tutorial for 45 minutes, they should shift their attention to the challenge problem. They are then shown a Jacob's ladder demonstration. After calculating voltage on the tutorial, we want them to estimate how large of a potential difference would actually be required to see sparks (15,000 V). More importantly, they should see that what they've just been working on can be applied to the real world. Many interesting questions came up during this demo, and it provided a very rich discussion (see comments below).

Reflection after administering the tutorial

Number of students: 12 (3 groups of 4)

Date: Friday, February 1. Third week of class.

My overall reaction after the tutorial was… that students wanted the tutorial to have more structure. At this point in their formal education, they have been trained to get the "right" answer. When they were asked to design their own field, they realized that there are many correct answers, so they easily lost focus on the task. I think that, in concept, this is a great activity. Unfortunately, since there was not one "magic" answer, students did not have an ultimate goal to motivate them.

Also, since this tutorial was a different format than previous ones, some groups were a little confused. One group didn't realize that they were supposed to work with the same E-field the whole way through. Because the format of this tutorial lacked structure, it might be a good idea to give more thorough directions for some problems.

At the end of the tutorial, a question was posed to several students that generated a lot of good discussion. The problem was a sphere with constant charge density, and V=0 far away. They were asked what the potential difference was between r=0 and r=infinity. There was a lot of "it must be zero" at first, which led to good discussion about continuity and the connection between E and V. This might be a good activity to replace (parts) of this tutorial in the future.

Part 1 – "Intelligently Designing" an Electric Field

i. Create a simple (but not trivial) Electric field. Your E-field must have a non-zero divergence ($\nabla \bullet \vec{E} \neq 0$) somewhere.

One student asked me, "I don't understand this restriction (that the divergence cannot be zero)… because if it is zero, that means there's no charge. If there's no charge, how can there be an E-field?" I replied that that was exactly the point, and what he said was exactly what we wanted him to see.

One group wasn't clear that they were supposed to write the E-field mathematically. Instead, they just drew some sketches. Another group wrote down a field that had a divergence, but also had a curl. I was happy to see that, because later in the tutorial they would have to confront it.

ii. Sketch your E-field. What charge distribution would cause this field?

Written by: Darren Tarshis
Contact: Steve Pollock (steven.pollock@colorado.edu)

No big problems (that I saw) here. However, for the group that drew their field on the previous questions, it made them curious why they were asked to draw it again.

iii. Find the E-field flux through the surface of a closed volume (of your choice). Can you give a reason why you get a positive, negative, or zero answer?

Students have had quite a bit of practice using Gauss's Law on homework, and in the last tutorial. So the only tricky part about this was choosing the "Best" shape (i.e. the one that would made their integral the easiest).

iv. Static E-fields have zero curl, $\nabla \times \vec{E} = 0$. If needed, modify yours so that it satisfies this requirement. But make sure it still satisfies the previous requirement (i.e. it must still have a non-zero divergence ($\nabla \bullet \vec{E} \neq 0$), somewhere).

One groups needed to modify their E-field to meet this requirement, but the others groups' fields already satisfied this. It was not clear to one group if they were supposed to use the same field they started with.

(This constraint is relevant on HW4Q3)

v. Is your E-field physically plausible? Is any E-field that has $\nabla \times \vec{E} = 0$ and $\nabla \bullet \vec{E} \neq 0$ physically plausible? If not, can you give an example of one that satisfies these two requirements, but is not realistic?

Some good discussion came from this. Students eventually distinguished between mathematically plausible, and physically possible. The issue of infinite charge emerged in the discussion. I told one student that at an infinite radius, his field would require an infinite amount of charge. He replied, "Well, you can't go to an infinite radius either." I wasn't sure how to respond to this.

(You will also determine if an E-field is physically realizable on HW4Q2b)

Part 2 – Measuring Electric Potential

If you had a multimeter, you could measure potential differences between any two points in space (if you had long enough leads).

Written by: Darren Tarshis
Contact: Steve Pollock (steven.pollock@colorado.edu)

i. Calculate the potential difference you would expect to measure between two arbitrary points. Choose two different paths to integrate over, and compare these answers.

One group realized that doing this integral with the E-field they had chosen (x-hat divided by script-r-hat cubed) would be too hard. So, they just changed their E-field (to x-hat divided by x cubed). Most students already understood that voltage was path independent –or at least they had memorized the phrase. I didn't see any students attempt two different integrals.

(You are asked to find Voltage from E-field on HW4Q5)

iii. If you had a really long lead, find the potential difference between the origin and far, far away. Why would someone choose far, far away as a reference point for your E-field?

One group of students had an E-field ~1/r, and they didn't know why they couldn't use infinity as a reference point. They saw that their field should go to zero at infinite radius, so it didn't make sense to them why their voltage integral was infinite. I had them think about a point charge, and they already knew that E drops off ~1/r^2. They did another integral using 1/r^2, and saw from the math that infinity works as a reference point. Two of the group members were very satisfied after seeing this from the math, realizing their function just didn't drop off fast enough. One group member still wasn't happy, and still didn't understand why he couldn't use infinity as a reference because when he plugged in infinity, his E~1/r was zero.

Challenge problem:

A Jacob's ladder is a continuous spark that rises between two conductors. (The dielectric strength of air is about three million Newtons per Coulomb)

- Estimate the voltage required for a Jacob's ladder to spark.
- Since it's plugged into the wall (120V AC), how is the voltage you estimated possible?
- What is the potential difference between the two conductors when they are closest together? What about when they are farthest apart?
- What is the potential difference between the very top of one conductor, and the very bottom of that same conductor (assuming it's about a meter long).
- Why does the spark rise?

• Why do you hear it?

The voltage estimate was not too difficult for the students, and it was nice that as a check, the voltage was printed on the Jacob's ladder. Almost everyone knew about the step-up transformer. Some of the students knew why the spark rises, but used very technical terms to explain it. If I could go back, I would ask the same students to describe it to me in the same way they would describe it to a 4th grader, that way I could figure out if they really understood, or if they were hiding behind fancy scientific terms (i.e. ionized plasma). I'm still not convinced that every student understands why the spark rises. One student gave a great reason why it makes sound: the spark heats the air, and pressure is proportional to temperature, so the change in pressure is what you hear as sound.

Relevant homework problems:

Q2. DIVERGENCE AND CURL

Consider an electric field $\mathbf{E} = c\dfrac{\vec{\mathbf{r}}}{r^2}$ (Please note the numerator is not $\hat{\mathbf{r}}$: this is NOT the usual E field from a point charge at the origin, which would give $c'\dfrac{\vec{\mathbf{r}}}{r^3}$, right?!)

a) - Calculate the divergence *and* the curl of this **E** field.

- Explicitly test your answer for the divergence by using the divergence theorem. (Is there a delta function at the origin like there was for a point charge field, or not?)
- Explicitly test your answer for the curl by using the formula given in Griffiths problem 1.60b, page 56.

b) What are the units of c? What charge distribution would you need to produce an E field like this? Describe it in words as well as formulas. (Is it physically realizable?)

Q3. ALLOWED E FIELDS

Which of the following two static E-fields is physically *impossible*. Why?

i) $\mathbf{E} = c(2x\hat{\mathbf{i}} - x\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

ii) $\mathbf{E} = c(2x\hat{\mathbf{i}} + z\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

where c is a constant (with appropriate units)

For the one which IS possible, find the potential V(r), using the *origin* as your reference point (i.e. setting V(0)=0)

- Check your answer by explicitly computing the gradient of V.

Written by: Darren Tarshis
Contact: Steve Pollock (steven.pollock@colorado.edu)

Note: you must select a specific path to integrate along. It doesn't matter which path you choose, since the answer is path-independent, but you can't compute a line integral without having a particular path in mind, so be explicit about that in your solution.

Q5. CALCULATING VOLTAGE FROM E FIELD

Last week, we investigated the electric field outside an infinite line that runs along the z-axis, $\vec{\mathbf{E}} = \frac{2\lambda}{4\pi\varepsilon_0}\frac{\vec{\mathbf{s}}}{s^2}$.

a) This field may look similar to Q2 above, but it is *not* the same - how is it different?
- Find the potential V(s) for points a distance "s" away from the z-axis.
(Note: you will have to have to be very careful to compute a difference of potentials between two points, or something similar, to avoid integrals which are infinite! You'll discuss this in part b)
- Check your answer by explicitly taking the gradient of V to make sure it gives you **E**.

b) Briefly discuss the question of "reference point": where did you set V=0? Can you use s=∞, or s=0, as the reference point, V(s)=0, here?
- How would your answer change if I told you that I wanted you to set V=0 at a distance s=3 meters away from the z-axis?
- Why is there trouble with setting V(∞)=0? (our usual choice), or V(0)=0 (often our second choice).

c) A typical Colorado lightning bolt might transfer a few Coulombs of charge in a stroke. Although lightning is clearly not remotely "electrostatic", let's pretend it is - consider a brief period during the stroke, and assume all the charges are fairly uniformly distributed in a long thin line. If you see the lightning stroke, and then a few seconds later hear the thunder, make a *very rough* estimate of the resulting voltage difference across a distance the size of your heart. (For you to think about - why is this not worrisome?)
What's the model? I am thinking of a lightning strike as looking rather like a long uniform line of charge... You've done the "physics" of this in the previous parts! (But e.g., you need a numerical estimate for λ. How long might that lightning bolt be? For estimation problems, don't worry about the small details, you can be off by 3, or even 10, I just don't want you off by factors of 1000's!)

Written by: Darren Tarshis
Contact: Steve Pollock (steven.pollock@colorado.edu)