





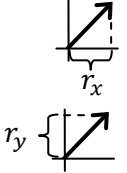


Word	Bra-ket	$\hat{i}$ & $\hat{j}$ Vectors	Matrix	Arrows
Vector	$ r\rangle$	$\vec{r} = r_x\hat{i} + r_y\hat{j}$	$\begin{pmatrix} r_x \\ r_y \end{pmatrix}$	
Adjoint Vector	$\langle r $	$\vec{r}^\dagger = r_x^*\hat{i} + r_y^*\hat{j}$	$(r_x^* \ r_y^*)$	
Ket Basis	$ 1\rangle$	$\hat{i}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	
	$ 2\rangle$	$\hat{j}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	
Bra Basis	$\langle 1 $	$\hat{i}$	$(1 \ 0)$	
	$\langle 2 $	$\hat{j}$	$(0 \ 1)$	
Orthogonality	$\langle 1 2\rangle$	$\hat{i} \cdot \hat{j}$	$(1 \ 0) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$	
Normalization	$\langle 1 1\rangle$	$\hat{i} \cdot \hat{i}$	$(1 \ 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$	
	$\langle 2 2\rangle$	$\hat{j} \cdot \hat{j}$	$(0 \ 1) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1$	
Length (Real)	$\langle r r\rangle = (r_x\langle 1  + r_y\langle 2 )(r_x 1\rangle + r_y 2\rangle)$ $= r_x^2\langle 1 1\rangle + r_xr_y\langle 1 2\rangle$ $+ r_yr_x\langle 2 1\rangle + r_y^2\langle 2 2\rangle$ $= r_x^2 + r_y^2$	$\vec{r} \cdot \vec{r} = (r_x\hat{i} + r_y\hat{j}) \cdot (r_x\hat{i} + r_y\hat{j})$ $= r_x^2\hat{i} \cdot \hat{i} + r_xr_y\hat{i} \cdot \hat{j}$ $+ r_yr_x\hat{j} \cdot \hat{i} + r_y^2\hat{j} \cdot \hat{j}$ $= r_x^2 + r_y^2$	$(r_x \ r_y) \begin{pmatrix} r_x \\ r_y \end{pmatrix} = r_x^2 + r_y^2$	
Components (Real)	$\langle 1 r\rangle = \langle 1 (r_x 1\rangle + r_y 2\rangle) = r_x\langle 1 1\rangle + r_y\langle 1 2\rangle = r_x$ $\langle 2 r\rangle = \langle 2 (r_x 1\rangle + r_y 2\rangle) = r_x\langle 2 1\rangle + r_y\langle 2 2\rangle = r_y$	$\hat{i} \cdot \vec{r} = \hat{i} \cdot (r_x\hat{i} + r_y\hat{j}) = r_x\hat{i} \cdot \hat{i} + r_y\hat{i} \cdot \hat{j} = r_x$ $\hat{j} \cdot \vec{r} = \hat{j} \cdot (r_x\hat{i} + r_y\hat{j}) = r_x\hat{j} \cdot \hat{i} + r_y\hat{j} \cdot \hat{j} = r_y$	$(1 \ 0) \begin{pmatrix} r_x \\ r_y \end{pmatrix} = r_x$ $(0 \ 1) \begin{pmatrix} r_x \\ r_y \end{pmatrix} = r_y$	
Inner Product (Real)	$\langle s r\rangle = (s_x\langle 1  + s_y\langle 2 )(r_x 1\rangle + r_y 2\rangle)$ $= s_xr_x\langle 1 1\rangle + s_xr_y\langle 1 2\rangle$ $+ s_yr_x\langle 2 1\rangle + s_yr_y\langle 2 2\rangle$ $= s_xr_x + s_yr_y$	$\vec{s} \cdot \vec{r} = (s_x\hat{i} + s_y\hat{j}) \cdot (r_x\hat{i} + r_y\hat{j})$ $= s_xr_x\hat{i} \cdot \hat{i} + s_xr_y\hat{i} \cdot \hat{j}$ $+ s_yr_x\hat{j} \cdot \hat{i} + s_yr_y\hat{j} \cdot \hat{j}$ $= s_xr_x + s_yr_y$	$(s_x \ s_y) \begin{pmatrix} r_x \\ r_y \end{pmatrix} = s_xr_x + s_yr_y$	

Word	Bra-ket	$\hat{i}$ & $\hat{j}$ Vectors	Matrix	Arrows
<b>Length (Complex)</b>	$\begin{aligned} \langle r r \rangle &= (r_x^* \langle 1  + r_y^* \langle 2 )(r_x  1\rangle + r_y  2\rangle) \\ &= r_x^* r_x \langle 1 1\rangle + r_x^* r_y \langle 1 2\rangle \\ &\quad + r_y^* r_x \langle 2 1\rangle + r_y^* r_y \langle 2 2\rangle \\ &= r_x^* r_x + r_y^* r_y =  r_x ^2 +  r_y ^2 \end{aligned}$		$(r_x^* \quad r_y^*) \begin{pmatrix} r_x \\ r_y \end{pmatrix} =  r_x ^2 +  r_y ^2$	
<b>Components (Complex)</b>	$\begin{aligned} \langle 1 r \rangle &= \langle 1 (r_x  1\rangle + r_y  2\rangle) = r_x \langle 1 1\rangle + r_y \langle 1 2\rangle \\ &= r_x \\ \langle 2 r \rangle &= \langle 2 (r_x  1\rangle + r_y  2\rangle) = r_x \langle 2 1\rangle + r_y \langle 2 2\rangle \\ &= r_y \end{aligned}$		$\begin{aligned} (1 \quad 0) \begin{pmatrix} r_x \\ r_y \end{pmatrix} &= r_x \\ (0 \quad 1) \begin{pmatrix} r_x \\ r_y \end{pmatrix} &= r_y \end{aligned}$	
<b>Inner Product (Complex)</b>	$\begin{aligned} \langle s r \rangle &= (s_x^* \langle 1  + s_y^* \langle 2 )(r_x  1\rangle + r_y  2\rangle) \\ &= s_x^* r_x \langle 1 1\rangle + s_x^* r_y \langle 1 2\rangle \\ &\quad + s_y^* r_x \langle 2 1\rangle + s_y^* r_y \langle 2 2\rangle \\ &= s_x^* r_x + s_y^* r_y \end{aligned}$		$(s_x^* \quad s_y^*) \begin{pmatrix} r_x \\ r_y \end{pmatrix} = s_x^* r_x + s_y^* r_y$	