SUPPLEMENT 1

ENERGY AND SPECTRAL POSITION UNITS AND INTERVALS

A review of some concepts in chapters 1 and 2 of the textbook (in particular p. 2 &16)

1. Energy or Spectral Position

You should know the following ways to express photon energy or spectral position and how to interconvert among them:

quantity	units
energy (E)	Joules, ergs, or electron volts
wavelength (λ)	nanometers, micrometers, or Angstroms
frequency (ν)	Hz (s^{-1})
wavenumber ($\bar{\nu}$)	cm^{-1}

example: for photons at 300 nm

$E = h\nu = 6.63 \times 10^{-34}$ J s x 1.00×10^{15} Hz = 6.63×10^{-19} J

= 6.63×10^{-19} J x (10^{7} ergs/J) = 6.63×10^{-12} ergs = 4.14 eV

2. Conversions between wavelength intervals and frequency intervals

carefully read - these concepts are often misunderstood

$$\Delta\nu = \nu_2 - \nu_1 = c/\lambda_2 - c/\lambda_1 \quad (1)$$

example: a 1-nm wavelength interval center at 300 nm

calculate ν_1 & ν_2: 299.5 nm equals 1.00167×10^{15} Hz

300.5 nm equals 0.99834×10^{15} Hz

so $\Delta\nu = 3.33 \times 10^{12}$ Hz

In many situations, one can also use the formula

$$\Delta\nu = c\Delta\lambda/\lambda^2 \quad (2)$$

where c/λ^2 is the conversion factor

so, $\Delta\nu = 3 \times 10^{8}$ m s^{-1} x 1×10^{-9} m/(300×10^{-9} m)^2 = 3.33×10^{12} Hz

which is the same answer as above. The approximation of equation 2 often works because from equation 1, $\Delta\nu = c(\lambda_1 - \lambda_2)/\lambda_1\lambda_2 \approx c\Delta\lambda/\lambda^2$ if $\lambda_2 \approx \lambda_1$ (i.e., $\lambda \approx (\lambda_2 + \lambda_1)/2$).

Equation 2 should only be used if $\Delta\lambda < 10$ nm. For example, consider a 100-nm interval centered at 300 nm, $\Delta\nu$ from equation 1 is 3.43×10^{14} Hz, but formula 2 yields 3.33×10^{14} Hz.

Equation 2 is often used to calculate atomic line widths which are typically 1 to 100 pm. This application will be discussed later (e.g., chapter 7 in textbook).

For practice fill out the rest of the table below and note the differences in the value of $\Delta\nu$ calculated with the two formulas.

		calculated from data in columns 1 and 2	calculated from equation 2
λ_1 = 200 nm	ν_1 =	$\Delta\lambda$ =	
λ_2 = 400 nm	ν_2 =	$\Delta\nu$ =	$\Delta\nu$ =

A common mistake to assume that conversion between a wavelength interval and a frequency interval is independent of the wavelength or frequency (e.g., incorrectly assume that $\Delta\nu = c/\Delta\lambda$). Because of the reciprocal relationship between wavelength and frequency, the conversion factor is proportional to λ^{-2}. Hence, for a 1-nm wavelength interval centered at 600 nm, the corresponding frequency interval is 8.33 x 10^{11} Hz or 1/4 of that at 300 nm.

3. Spectral Radiometric Quantities

Remember the **spectral radiometric quantities** have units including per nanometer or per Hz (the nanometer or Hertz represents an interval not an absolute value). Thus, the conversion factor between the two quantities is the reciprocal of that for equation 2 as shown for spectral radiant power below

$$\Phi_\nu = \Phi_\lambda \lambda/\nu = \Phi_\lambda \lambda^2/c$$

where the conversion factor has units of nm/Hz. Hence, if the spectral radiance of a source in terms of nanometers is the same at 300 and 600 nm, the spectral radiance in terms of Hertz is four times greater at 600 nm compared to that at 300 nm. This occurs because a 1-Hz interval at 600 nm corresponds to four times the wavelength interval at 300 nm.

Conversion between units in terms of Watts to in terms of photons per second should also be second nature. Each photon has an energy of hν in Joules so photons/s times joules/photon is joules/s or Watts. Hence to convert any radiometric quantity in terms of Watts per whatever (e.g., W/sr) to photons/s per whatever, just divide by $h\nu$. Likewise, intensities in terms of photons/s are converted to Watts by multiplying by $h\nu$. **For conversion between irradiance and power density**, see footnote of Table 2-1.

Practice problems: 1. What is the frequency of 600 nm light? (ans: 5.0×10^{14} Hz); 2. A lamp has a radiance of 1.0 W/cm^2-sr-nm at 300 nm. What is the radiance at this wavelength in units of per Hz? (ans: 3.0×10^{-13} W/cm^2-sr-Hz). 3. What is the radiant power in Watts for a beam of 1×10^6 photons/s (ans: 6.6×10^{-13} W)

Addition to supplement 1

Intensity calculations and working with radiometric units (pages 15 to 18 in the textbook).

Many practical calculations dealing with intensity and throughput in spectrometers involve calculating how many photons get from point a to point b. More specifically, one wants to know how much light (radiant power) emitted by a source (light source such as a tungsten lamp, atomic source such as a plasma, or a solution fluorescing in a sample cell) is transferred to some receptor (mirror, lens, photodetector) or aperture (slit). Below are some guidelines about such calculations:

1. Determine if the source is a **point, extended, or collimated source**. A laser produces collimated radiation and conventional sources such as tungsten lamps are normally used as extended sources where only a portion of the source is actually viewed as determined by some aperture.
2. For a point source, the preferred unit is **radiant intensity (I)** and it is calculated as the total radiant power emitted in all directions divided by 4π. The radiant reaching a receptor is $I\Omega$ where **Ω is the solid angle** viewed by the receptor.
3. For an extended source, the preferred unit is **radiance (B)** and it is calculated as the total radiant power emitted in all directions divided by the product of 4π and the projected area. The **projected area** is the apparent area as viewed from a distance. Hence, the projected area is a square for a cube source and a circle for a spherical source. If the source (or receptor) is a flat surface the projected area is the actual area times the cosine of the angle defined by the normal to the surface and line between the source and receptor. The radiant reaching a receptor is $BA_s\Omega$ where **A_s is the area of the source viewed** (as determined by some aperture).
4. The **irradiance (W/cm^2)** incident on a receptor is calculated at the total radiant power striking the receptor divided by the projected area of the receptor (A_r). Be careful not to confuse the source area and aperture with the receptor area or aperture.
5. To determine the radiant power striking a receptor one must know the **distance between the source and receptor (d)**, the projected area of the receptor, and the area of the source viewed if it is extended. The distance and projected area allow one to calculate the solid viewed by the receptor as the projected area divided by the distance squared.
6. The term **"limiting aperture"** is common. The limiting aperture is what limits the solid angle or sometimes the area viewed between a source and receptor. Sometimes the limiting aperture is the receptor itself such as the dimension of a mirror or lens. There could also be a physical opening

between the source and receptor that limits rays of light that can reach the receptor.