Proofs for Ch 553

The following are a set of thermodynamic identities,

$$\left(\frac{\partial T}{\partial P}\right)_H = -\frac{1}{C_p}\left(\frac{\partial H}{\partial P}\right)_T \tag{1}$$

$$C_v = T\left(\frac{\partial S}{\partial T}\right)_V \tag{2}$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \tag{3}$$

$$\left(\frac{\partial A}{\partial V}\right)_S = \frac{\alpha_p T S}{\kappa_T C_v} - P \tag{4}$$

$$\left(\frac{\partial (1/T)}{\partial P}\right)_H = \frac{1}{T^2 C_p}\left(V - T\left(\frac{\partial V}{\partial T}\right)_P\right) \tag{5}$$

$$\rho\left(\frac{\partial \mu}{\partial \rho}\right)_T = \left(\frac{\partial P}{\partial \rho}\right)_T \tag{6}$$

$$\left(\frac{\partial U}{\partial P}\right)_T = V(\kappa_T P - \alpha_p T) \tag{7}$$

$$\left(\frac{\partial H}{\partial T}\right)_V = C_p\left(1 - \frac{\alpha_p \mu_{JT}}{\kappa_T}\right) \tag{8}$$

$$C_p(\kappa_T - \kappa_S) = TV\alpha_p^2 \tag{9}$$

$$\left(\frac{\partial A}{\partial P}\right)_S = V\left(\frac{\kappa_T P}{\gamma} - \frac{\alpha_P T S}{C_p}\right) \tag{10}$$

$$\left(\frac{\partial T}{\partial P}\right)_S = TV\frac{\alpha_p}{C_p} \tag{11}$$

$$\frac{C_p}{C_v} = \frac{\kappa_T}{\kappa_S} \tag{12}$$

$$C_p = C_v + T\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_V \tag{13}$$

where α_P is the coefficient of thermal expansion, κ_T is the isothermal compressibility, κ_S is the adiabatic compressibility (S held constant), $\gamma = C_p/C_v$ and $\rho = n/V$.

Eqs(1-10) are for your benefit. Enjoy yourself. To receive credit, you will derive Eqs(11-13) in my office during 5th or 6th week of term, prior to the exam. Fifteen minutes are allotted for all three proofs. The proofs are 'worth' one problem set. No assistance will be provided during your proof and you are not allowed to use paper, notes, books etc. You can assume the equations for $dU(S,V)$, and the defining relations for H, G and A, but from that point onward, all results must be derived. On what constitutes a proof: you will start with the left hand side of the equation and not touch the right hand side.