

Worksheet # 5

Friday, February 2, 2024

Name**Questions (5 pts):**

As we discussed last time, for the matrices $R_{\mathbf{n}}$ representing geometrical rotations around an axis \mathbf{n} by a small angle ε the following relation is valid:

$$R_x(\varepsilon)R_y(\varepsilon) - R_y(\varepsilon)R_x(\varepsilon) = R_z(\varepsilon^2) - 1$$

Using this relation, as well as direct correspondence between $R_{\mathbf{n}}$ and the QM rotation operator $D(\mathbf{n},\varphi)$ and the fact that for small angles ε the operator $D(\mathbf{n},\varepsilon)$ is:

$$D(\mathbf{n},\varepsilon) \approx 1 - (i/\hbar) (\mathbf{J} \cdot \mathbf{n})\varepsilon + (1/2)(i/\hbar)^2 ((\mathbf{J} \cdot \mathbf{n})\varepsilon)^2,$$

derive the commutation relations for the angular momentum operators J_i .