Physics 652 Winter 2024

Homework #8

(due Wednesday, March 13, 2024)

1. (10 pts) In class we obtained the expansion of the perturbed eigenstate $|n\rangle$ to the first order in λ. Derive the expression for the perturbed eigenstate to the second order in λ, i.e. find $|n^{(2)}\rangle$.

2. (10 pts) Consider two-dimensional harmonic oscillator perturbed with the potential $V = \alpha xy$. Find the energy splitting for the lowest degenerate state.

3. (20 pts) Consider a spin-1 particle, whose Hamiltonian is given by $H = AS_z^2 + B\left(S_x^2 - S_y^2\right)$,where

$$S_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}; \quad S_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}; \quad S_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

, and A, B are positive constants, $A \gg B$.

(a) Find the exact solution of the Schroedinger equation, i.e. determine the energy levels and eigenstates.

(b) Now treat the same Hamiltonian as $H = H_0 + V = AS_z^2 + B\left(S_x^2 - S_y^2\right)$, so that $H_0 = AS_z^2$ is the unperturbed Hamiltonian, and $V = B(S_x^2 - S_y^2)$ is a perturbation. **Note: in this part, you do not need to use the matrix representation of the operators S_i, which you used in part (a). Use your knowledge of the properties of angular momentum. In particular:**

(i) Use $|s, m_s\rangle$ basis to find the energy levels of the *unperturbed* system. You should obtain one non-degenerate level and one double-degenerate level.

(ii) Find the first order energy correction for the non-degenerate level.

(iii) Find the first order energy corrections for the degenerate level (in particular, show that the perturbation removes degeneracy) and corresponding eigenstates.

(iv) Compare your results with the exact solution you found in part (a).

4. Reading assignment: Sakurai 5.1, 5.2.