Physics 652 Winter 2024

Homework #4

(due Wednesday, February 7, 2024)

1. (10 pts) For the hydrogen atom, find the expectation value of 1/r (r is the distance between the particles) in the state $|n,l,m\rangle$.

2. **(**20 pts) Find the number of *s* bound states for a particle of mass *m* moving in a potential $V(r) = -V_0\,\delta(r-a)$, where $V_0 > 0$. Discuss the existence of bound states in terms of the size of *a*. Find the normalized wave function of the bound state(s).

3. (10 pts) Show that $(\boldsymbol{\sigma}\cdot\mathbf{a})(\boldsymbol{\sigma}\cdot\mathbf{b}) = \mathbf{a}\cdot\mathbf{b} + i\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b})$, where **a**, **b** are arbitrary vectors, and **σ**'s are the Pauli matrices.

4. (10 pts) Using the Pauli matrices σ_i, show:
   (a) $\exp[-i\alpha\sigma_x] = I\cos\alpha - i\,\sigma_x \sin\alpha$, where I is a unit matrix.
   (b) $\exp[i\alpha\sigma_x]\,\sigma_z \exp[-i\alpha\sigma_x] = \sigma_z\cos(2\alpha) + \sigma_y \sin(2\alpha)$

5. Reading: Sakurai 3.1-3.3