

Homework #2

(due Wednesday, October 11, 2023)

1. (20 pts) In the following expressions, where A is an operator, specify the nature of each expression (i.e. whether it's an operator, a bra or a ket) and then find its Hermitian conjugate:

(a) $\langle \psi | A | \phi \rangle \langle \phi |$

(b) $A | \phi \rangle \langle \psi |$

(c) $\langle \psi | A | \phi \rangle | \phi \rangle \langle \psi | A$

(d) $\langle \phi | A | \psi \rangle | \phi \rangle - iA | \psi \rangle$

(e) $| \phi \rangle \langle \phi | (A + iA) | \psi \rangle \langle \psi |$

2. (15 pts) In class, we looked at the “hermiticity” of the operators X , d/dx and $-i\hbar(d/dx)$.

- (a) Use our results to explore whether the following operators are Hermitian

$$e^X, e^{d/dx} \text{ and } e^{-i\hbar d/dx}.$$

- (b) Find the Hermitian conjugate of the operator $X(d/dx)$, where X is a position operator. Present your result as $XA+B$, where A and B are some operators. What are these operators A and B ?

3. (20 pts) Consider an operator $A = i(X^2 + 1)d/dx + iX$

- (a) Show that A is Hermitian

- (b) Find the normalized state $\psi(x)$, where x spans from $-\infty$ to $+\infty$, for which

$$A\psi(x) = 0.$$

- (c) Calculate the probability of finding the particle (represented by $\psi(x)$) in the region $-1 \leq x \leq 1$.

4. Reading assignment: Sakurai 1.2